EEL 3473 - Friday (1)

HW # 7 SOLUTIONS Oct. 23, 2009

Problem 8.26 A plane wave in air with

$$\widetilde{\mathbf{E}}^{i} = \hat{\mathbf{y}}20e^{-j(\underbrace{3x}_{k_x x}+\underbrace{4z}_{k_z z})} \quad \text{(V/m)},$$

$k_x = 3 = k_1 \sin\theta_i$
$k_y = 4 = k_1 \cos\theta_i$

see class notes

is incident upon the planar surface of a dielectric material, with $\varepsilon_r = 4$, occupying the half space $z \geq 0$. Determine:

(a) the polarization of the incident wave,
(b) the angle of incidence,
(c) the time-domain expressions for the reflected electric and magnetic fields,
(d) the time-domain expressions for the transmitted electric and magnetic fields, and
(e) the average power density carried by the wave in the dielectric medium.

Solution:

(a) $\widetilde{\mathbf{E}}^{i} = \hat{\mathbf{y}}20e^{-j(3x+4z)}$ V/m.

Since $\mathbf{E}^i$ is along $\hat{\mathbf{y}}$, which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) ~~From Eq. (8.48a)~~, the argument of the exponential is $-j(k_x x + k_z z) = -j(k_1 \sin\theta_i x + k_1 \cos\theta_i z)$

$$\cancel{-jk_1(x\sin\theta_i + z\cos\theta_i)} = -j(3x+4z).$$

$\widetilde{\vec{E}} = \hat{y}\,20e^{-j\vec{k}\cdot\vec{r}}$

Hence,

$$k_1 \sin\theta_i = 3, \qquad k_1 \cos\theta_i = 4,$$

from which we determine that

$$\tan\theta_i = \frac{3}{4} \quad \text{or} \quad \theta_i = 36.87^\circ,$$

$\vec{r} = x\hat{x} + z\hat{z}$
$\vec{k}_1 = k_x\hat{x} + k_z\hat{z}$
$\vec{k}_1 = 3\hat{x} + 4\hat{z}$

and

$$k_1 = \sqrt{\underbrace{3^2}_{k_x} + \underbrace{4^2}_{k_z}} = 5 \quad \text{(rad/m)}.$$

$|\vec{k}_1| = \sqrt{k_x^2 + k_z^2}$

Also,

$$\omega = u_p k = ck = 3 \times 10^8 \times 5 = 1.5 \times 10^9 \quad \text{(rad/s)}.$$

(c)

$$\eta_1 = \eta_0 = 377\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{\eta_0}{2} = 188.5\ \Omega,$$

$$\theta_t = \sin^{-1}\left[\frac{\sin\theta_i}{\sqrt{\varepsilon_{r_2}}}\right] = \sin^{-1}\left[\frac{\sin 36.87^\circ}{\sqrt{4}}\right] = 17.46^\circ,$$

ẑ

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -0.41,$$
$$\tau_\perp = 1 + \Gamma_\perp = 0.59.$$

In accordance with Eq. (8.49a), and using the relation $E_0^r = \Gamma_\perp E_0^i$,

$$\widetilde{\mathbf{E}}^r = -\hat{\mathbf{y}}\, 8.2 e^{-j(3x-4z)},$$
$$\widetilde{\mathbf{H}}^r = (\hat{\mathbf{x}} \cos\theta_i + \hat{\mathbf{z}} \sin\theta_i)\, \frac{8.2}{\eta_0}\, e^{-j(3x-4z)},$$

where we used the fact that $\theta_i = \theta_r$ and the z-direction has been reversed.

$$\mathbf{E}^r = \Re\mathfrak{e}[\widetilde{\mathbf{E}}^r e^{j\omega t}] = -\hat{\mathbf{y}}\, 8.2 \cos(1.5\times 10^9 t - 3x + 4z) \quad \text{(V/m)},$$
$$\mathbf{H}^r = (\hat{\mathbf{x}}\, 17.4 + \hat{\mathbf{z}}\, 13.06) \cos(1.5\times 10^9 t - 3x + 4z) \quad \text{(mA/m)}.$$

(d) In medium 2,

$$k_2 = k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5\sqrt{4} = 20 \quad \text{(rad/m)},$$

and

$$\theta_t = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_i\right] = \sin^{-1}\left[\frac{1}{2} \sin 36.87^\circ\right] = 17.46^\circ$$

and the exponent of $\mathbf{E}^t$ and $\mathbf{H}^t$ is

$$-jk_2(x\sin\theta_t + z\cos\theta_t) = -j10(x \sin 17.46^\circ + z\cos 17.46^\circ) = -j(3x + 9.54z).$$

Hence,

$$\widetilde{\mathbf{E}}^t = \hat{\mathbf{y}}\, 20 \times 0.59 e^{-j(3x+9.54z)},$$
$$\widetilde{\mathbf{H}}^t = (-\hat{\mathbf{x}} \cos\theta_t + \hat{\mathbf{z}} \sin\theta_t)\, \frac{20 \times 0.59}{\eta_2}\, e^{-j(3x+9.54z)}.$$
$$\mathbf{E}^t = \Re\mathfrak{e}[\widetilde{\mathbf{E}}^t e^{j\omega t}] = \hat{\mathbf{y}}\, 11.8 \cos(1.5\times 10^9 t - 3x - 9.54z) \quad \text{(V/m)},$$
$$\mathbf{H}^t = (-\hat{\mathbf{x}} \cos 17.46^\circ + \hat{\mathbf{z}} \sin 17.46^\circ)\, \frac{11.8}{188.5} \cos(1.5\times 10^9 t - 3x - 9.54z)$$
$$= (-\hat{\mathbf{x}}\, 59.72 + \hat{\mathbf{z}}\, 18.78)\, \cos(1.5\times 10^9 t - 3x - 9.54z) \quad \text{(mA/m)}.$$

(e)

$$S^t_{av} = \frac{|E_0^t|^2}{2\eta_2} = \frac{(11.8)^2}{2 \times 188.5} = 0.36 \quad (\text{W/m}^2).$$