EEL 3473, HW 4 - Solution

Uman

$$\tilde{E}_x(z) = E_{x0}^{+} e^{-jkz}$$

$$k = \omega\sqrt{\mu_0\epsilon_0(1-\omega_{pe}^2/\omega^2)} \qquad , \omega_{pe}^2 = \frac{n_e|e|^2}{m_e\epsilon_0}$$

(1) $$\frac{E_{x0}^{+}}{H_{y0}^{+}} = \sqrt{\frac{\mu_0}{\epsilon_0(1-\omega_{pe}^2/\omega^2)}}$$

(2) $$v_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\epsilon_0(1-\frac{\omega_{pe}^2}{\omega^2})}}$$

(3) $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu_0\epsilon_0(1-\omega_{pe}^2/\omega^2)}}$$

valid for $\omega > \omega_{pe}$ and $\tilde{E}_x(z) = E_{x0}^{+} e^{-jkz}$

note that $v_{ph} > \frac{1}{\sqrt{\mu_0\epsilon_0}} = c$

For $\omega >> \omega_{pe}$ (electrons have no time to move, so wave thinks it is in free space)

(1) $$\frac{E_{x0}^{+}}{H_{y0}^{+}} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

(2) $$v_{ph} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

(3) $$\lambda = \frac{c}{f}$$

above equations in limit of high $\omega$

For $\omega < \omega_{pe}$

$$k = \pm j\omega\sqrt{\mu_0\epsilon_0\left(\frac{\omega_{pe}^2}{\omega^2}-1\right)}$$

and $$\tilde{E}_x = E_{x0}^{+} e^{-jkz} = E_{x0}^{+} e^{-\omega\sqrt{\mu_0\epsilon_0\left(\frac{\omega_{pe}^2}{\omega^2}-1\right)}\, z}$$

The wave ~~attenuates~~ without ohmic loss (so-called reactively) and without oscillation - it is not propagating (no phase term)

The expressions for velocity and wavelength are not of use since the wave is not propagating.

Plasma frequency is the natural freq. of oscillation of the electrons. The plasma will not support propagation below that frequency.