EEL 3473 Home Work # 3 Solutions
Fall 2009

Problem 7.1 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}}30\cos(10^8 t - 0.5y) \quad \text{(mA/m)}.$$

Find (a) the direction of wave propagation, (b) the phase velocity, (c) the wavelength in the material, (d) the relative permittivity of the material, and (e) the electric field phasor.

Solution:

(a) Positive y-direction, because of the minus sign in $(10^8 t - 0.5y)$

(b) $\omega = 10^8$ rad/s, $k = 0.5$ rad/m.

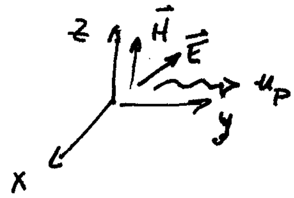
$$u_p = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \times 10^8 \text{ m/s}.$$

(c) $\lambda = 2\pi/k = 2\pi/0.5 = 12.6$ m.

(d) $\varepsilon_r = \left(\dfrac{c}{u_p}\right)^2 = \left(\dfrac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25.$

(e) From Eq. (7.39b),

$$\widetilde{\mathbf{E}} = -\eta\hat{\mathbf{k}} \times \widetilde{\mathbf{H}},$$ and $\vec{E} \times |H|\hat{z} \to +y$-direction so $\vec{E}$ in $-x$-direction



$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.5} = 251.33 \quad (\Omega),$$

$$\hat{\mathbf{k}} = \hat{\mathbf{y}}, \quad \text{and} \quad \widetilde{\mathbf{H}} = \hat{\mathbf{z}}30e^{-j0.5y} \times 10^{-3} \quad \text{(A/m)}.$$

Hence,

$$\widetilde{\mathbf{E}} = -251.33\hat{\mathbf{y}} \times \hat{\mathbf{z}}30e^{-j0.5y} \times 10^{-3} = -\hat{\mathbf{x}}7.54e^{-j0.5y} \quad \text{(V/m)},$$

and

$$\mathbf{E}(y,t) = \mathfrak{Re}(\widetilde{\mathbf{E}}e^{j\omega t}) = -\hat{\mathbf{x}}7.54\cos(10^8 t - 0.5y) \quad \text{(V/m)}.$$

Problem 7.4 The electric field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{E} = [\hat{\mathbf{y}}3\sin(\overbrace{\pi \times 10^7}^{\omega} t - \overbrace{0.2\pi}^{k} x) + \hat{\mathbf{z}}4\cos(\pi \times 10^7 t - 0.2\pi x)] \quad \text{(V/m)}.$$

Determine (a) the wavelength, (b) ε_r, and (c) $\mathbf{H}$.

Solution:

(a) Since $k = 0.2\pi$,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \text{ m}.$$

(b)

$$u_p = \frac{\omega}{k} = \frac{\pi \times 10^7}{0.2\pi} = 5 \times 10^7 \text{ m/s}.$$

But

$$u_p = \frac{c}{\sqrt{\varepsilon_r}}.$$

Hence,

$$\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3 \times 10^8}{5 \times 10^7}\right)^2 = 36.$$

(c)

$$\begin{aligned}\mathbf{H} &= \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E} = \frac{1}{\eta}\hat{\mathbf{x}} \times [\hat{\mathbf{y}}3\sin(\pi \times 10^7 t - 0.2\pi x) + \hat{\mathbf{z}}4\cos(\pi \times 10^7 t - 0.2\pi x)] \\ &= \hat{\mathbf{z}}\frac{3}{\eta}\sin(\pi \times 10^7 t - 0.2\pi x) - \hat{\mathbf{y}}\frac{4}{\eta}\cos(\pi \times 10^7 t - 0.2\pi x) \quad \text{(A/m)},\end{aligned}$$

with

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} \simeq \frac{120\pi}{6} = 20\pi = 62.83 \quad (\Omega).$$

Problem 7.14 For each of the following combination of parameters, determine if the material is a low-loss dielectric, a quasi-conductor, or a good conductor, and then calculate α, β, λ, u_p, and η_c:

(a) glass with $\mu_r = 1$, $\varepsilon_r = 5$, and $\sigma = 10^{-12}$ S/m at 10 GHz,

(b) animal tissue with $\mu_r = 1$, $\varepsilon_r = 12$, and $\sigma = 0.3$ S/m at 100 MHz,

(c) wood with $\mu_r = 1$, $\varepsilon_r = 3$, and $\sigma = 10^{-4}$ S/m at 1 kHz.

Solution: Using equations given in Table 7-1:

	Case (a)	Case (b)	Case (c)
$\sigma/\omega\varepsilon$	3.6×10^{-13}	4.5	600
Type	low-loss dielectric	quasi-conductor	good conductor
α	8.42×10^{-11} Np/m	9.75 Np/m	6.3×10^{-4} Np/m
β	468.3 rad/m	12.16 rad/m	6.3×10^{-4} rad/m
λ	1.34 cm	51.69 cm	10 km
u_p	1.34×10^8 m/s	0.52×10^8 m/s	0.1×10^8 m/s
η_c	$\simeq 168.5\ \Omega$	$39.54 + j31.72\Omega$	$6.28(1+j)\ \Omega$

Problem 7.15 Dry soil is characterized by $\varepsilon_r = 2.5$, $\mu_r = 1$, and $\sigma = 10^{-4}$ (S/m). At each of the following frequencies, determine if dry soil may be considered a good conductor, a quasi-conductor, or a low-loss dielectric, and then calculate α, β, λ, μ_p, and η_c:

(a) 60 Hz,

(b) 1 kHz,

(c) 1 MHz,

(d) 1 GHz.

Solution: $\varepsilon_r = 2.5$, $\mu_r = 1$, $\sigma = 10^{-4}$ S/m.

$f \rightarrow$	60 Hz	1 kHz	1 MHz	1 GHz
$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi f \varepsilon_r \varepsilon_0}$	1.2×10^4	720	0.72	7.2×10^{-4}
Type of medium	Good conductor	Good conductor	Quasi-conductor	Low-loss dielectric
α (Np/m)	1.54×10^{-4}	6.28×10^{-4}	1.13×10^{-2}	1.19×10^{-2}
β (rad/m)	1.54×10^{-4}	6.28×10^{-4}	3.49×10^{-2}	33.14
λ (m)	4.08×10^4	10^4	180	0.19
u_p (m/s)	2.45×10^6	10^7	1.8×10^8	1.9×10^8
η_c (Ω)	$1.54(1+j)$	$6.28(1+j)$	$204.28 + j65.89$	238.27

Problem 7.24 The inner and outer conductors of a coaxial cable have radii of 0.5 cm and 1 cm, respectively. The conductors are made of copper with $\varepsilon_r = 1$, $\mu_r = 1$ and $\sigma = 5.8 \times 10^7$ S/m, and the outer conductor is 0.5 mm thick. At 10 MHz:

(a) Are the conductors thick enough to be considered infinitely thick so far as the flow of current through them is concerned?

(b) Determine the surface resistance R_s.

(c) Determine the a-c resistance per unit length of the cable.

Solution:

(a) From Eqs. (7.72) and (7.77b),

$$\delta_s = [\pi f \mu \sigma]^{-1/2} = [\pi \times 10^7 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{-1/2} = 0.021 \text{ mm}.$$

Hence,

$$\frac{d}{\delta_s} = \frac{0.5 \text{ mm}}{0.021 \text{ mm}} \approx 25.$$

Hence, conductor is plenty thick.

(b) From Eq. (7.92a),

$$R_s = \frac{1}{\sigma \delta_s} = \frac{1}{5.8 \times 10^7 \times 2.1 \times 10^{-5}} = 8.2 \times 10^{-4} \ \Omega.$$

(c) From Eq. (7.96),

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{8.2 \times 10^{-4}}{2\pi}\left(\frac{1}{5 \times 10^{-3}} + \frac{1}{10^{-2}}\right) = 0.039 \quad (\Omega/\text{m}).$$

$\frac{|\vec{E}|}{|\vec{H}|} = 377$ $\eta_0 \hat{=} 120\pi \hat{=} 377\,\Omega$ $\times 10^{-6}$ since $\vec{E}$ is in mV/m

Problem 7.25 The magnetic field of a plane wave traveling in air is given by $\mathbf{H} = \hat{\mathbf{x}}50\sin(2\pi \times 10^7 t - ky)$ (mA/m). Determine the average power density carried by the wave.

Solution: $T = \frac{1}{f}$, also $S_{AV} = \frac{1}{T}\int_0^T \eta_0 |\vec{E}(t)|^2 dt = \frac{1}{T}\int_0^T 120\pi (50)^2 \sin^2(2\pi\times 10^7 t - ky)\,dt$ 10^{-6}

in time domain

since $\sin^2 x = \frac{1}{2} - \frac{1}{2}\sin 2x$ [2nd term integrates to zero

$S_{AV} = \frac{120\pi(50)^2}{2} \times 10^{-6}$ Watts/m^2

$$\mathbf{H} = \hat{\mathbf{x}}50\sin(2\pi \times 10^7 t - ky) \quad \text{(mA/m)},$$

$$\mathbf{E} = -\eta_0 \hat{\mathbf{y}} \times \mathbf{H} = \hat{\mathbf{z}}\eta_0 50\sin(2\pi \times 10^7 t - ky) \quad \text{(mV/m)},$$

$$\mathbf{S}_{\rm av} = (\hat{\mathbf{z}} \times \hat{\mathbf{x}})\frac{\eta_0(50)^2}{2} \times 10^{-6} = \hat{\mathbf{y}}\frac{120\pi}{2}(50)^2 \times 10^{-6} = \hat{\mathbf{y}}0.48 \quad \text{(W/m}^2\text{)}.$$

direction is $+y$ because of minus sign in $(-\hat{y}\,ky)$

Problem 7.26 A wave traveling in a nonmagnetic medium with $\varepsilon_r = 9$ is characterized by an electric field given by

$$\mathbf{E} = [\hat{\mathbf{y}}3\cos(\pi \times 10^7 t + kx) - \hat{\mathbf{z}}2\cos(\pi \times 10^7 t + kx)] \quad \text{(V/m)}.$$

Determine the direction of wave travel and the average power density carried by the wave.

Solution:

$$\eta \simeq \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$

The wave is traveling in the negative x-direction.

$$\mathbf{S}_{\rm av} = -\hat{\mathbf{x}}\frac{[3^2 + 2^2]}{2\eta} = -\hat{\mathbf{x}}\frac{13}{2 \times 40\pi} = -\hat{\mathbf{x}}0.05 \quad \text{(W/m}^2\text{)}.$$

not assigned
