TEST #4 Solutions

April 22, 2009

Problem 1.

Write Maxwell's Equations in point and in integral form.

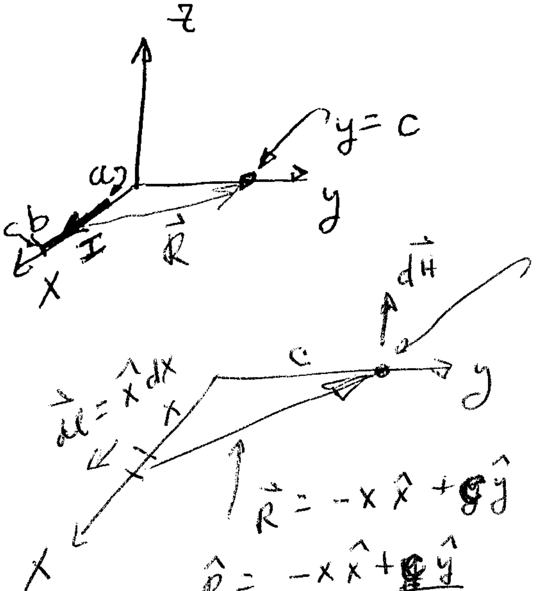
$$\nabla \cdot \vec{D} = \rho_v \qquad \oint\!\!\oint \vec{D} \cdot d\vec{s} = Q_{encl}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint\!\!\oint \vec{B} \cdot d\vec{s} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \oint \vec{H} \cdot d\vec{l} = \underbrace{\iint \vec{J} \cdot d\vec{s}}_{I_{encl}} + \underbrace{\iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}}_{I_{disp.\ encl.}}$$

Problem 2.
A current flows along the x-axis from x = a to x = b, as shown. Set up an expression which can be integrated for the magnetic flux density in free space on the y-axis at y = c. The expression should not contain cross-products. Show all steps.



$$\vec{R} = -x\hat{x} + c\hat{y}$$

$$\hat{R} = \frac{-x\hat{x} + c\hat{y}}{\sqrt{x^2+c^2}}$$

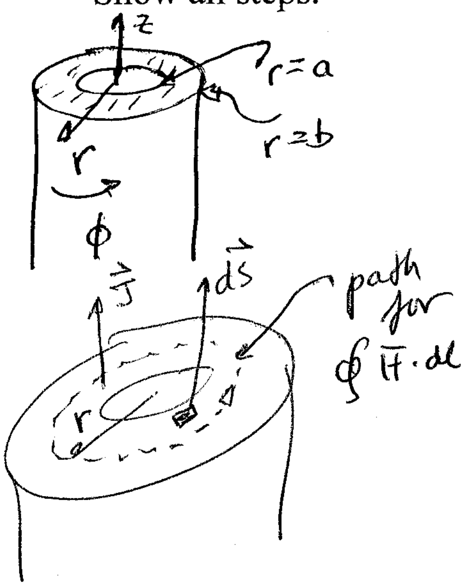
$$d\vec{H} = \frac{\underbrace{I\hat{x}\,dx}_{I\,d\vec{l}} \times \overbrace{\left(\frac{-x\hat{x} + c\hat{y}}{\sqrt{x^2+c^2}}\right)}^{\hat{R}}}{4\pi\underbrace{(x^2+c^2)}_{R^2}} = \frac{I c\,\hat{z}\,dx}{4\pi(x^2+c^2)^{3/2}}$$

$$\vec{H} = \hat{z}\int_{x=a}^{x=b} \frac{I c\,dx}{4\pi(x^2+c^2)^{3/2}}$$

$$\boxed{\vec{B} = \hat{z}\int_{x=a}^{b} \frac{\mu_0 I c\,dx}{4\pi(x^2+c^2)}}$$

Problem 3.

A hollow cylinder of thickness (b-a) and permeability μ, whose inner surface is located at r = a, is concentric with the z-axis and carries a current density $\vec{J} = \hat{z}(k/r)$. Find the magnetic flux density between r = a and r = b. Show all steps.



(1) argue that the only magnetic field is $H_\phi(R)$, from symmetry

(2) Apply Ampere Law to path at left

$$\oint \vec{H} \cdot d\vec{l} = \int_{\phi=0}^{2\pi} \int_{r'=a}^{r'=r} \underbrace{\hat{z} K / r'}_{\vec{J}} \cdot \underbrace{r' \, dr' \, d\phi \, \hat{z}}_{d\vec{S}}$$

$$\int_{\phi=0}^{2\pi} H_\phi(R) \hat{\phi} \cdot r \, d\phi \, \hat{\phi} = \underbrace{\int_{r'=a}^{r} \int_{\phi=0}^{2\pi} K \, d\phi \, dr'}_{\text{current enclosed within dotted line on dwg}}$$

$$\cancel{2\pi} r H_\phi(R) = \cancel{2\pi} (r-a) K$$

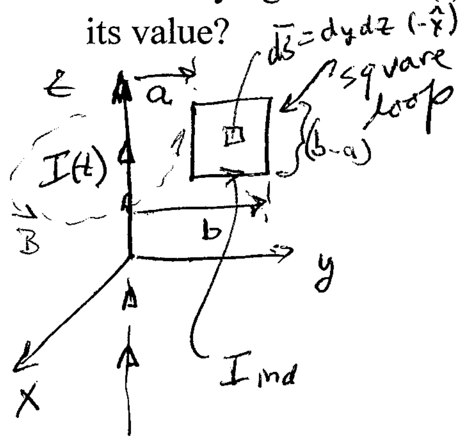
$$H_\phi(R) = \frac{K(r-a)}{r}$$

$$\bar{B} = \mu \bar{H}$$

$$\boxed{\vec{B} = \frac{\mu K (r-a)}{r} \hat{\phi} \ \text{Web}/m^2}$$

Problem 4.

A square conductive loop, with the geometry shown, lies in the y-z plane and has a resistance R. An infinitely long wire along the z-axis carries a time-varying current $I(t) = Kt^2$. Does current flow in the loop? If so, what is its value?



$$V_{\substack{emf\\ in\\ loop}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{ds}$$

$$\vec{B}_{through\ loop} = \frac{\mu_0 I(t)}{2\pi y}(-\hat{x}) \quad \left[\text{field of } \infty \text{ wire}\right] \text{ see below left}$$

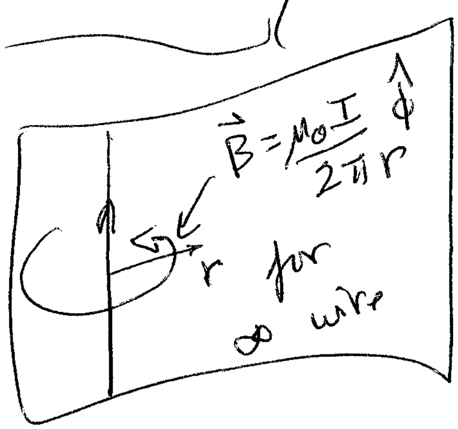
$$I_{induced} = \frac{V_{emf}}{R} = -\frac{1}{R}\frac{d}{dt}\int_{y=a}^{b}\int_{z=0}^{b-a} \overbrace{\frac{\mu_0 I(t)}{2\pi y}(-\hat{x})}^{\vec{B}}\cdot\underbrace{dy\,dz(-\hat{x})}_{\vec{ds}}$$

$\vec{B}$ increases through the loop into the page – Induced current tries to buck the increase

$$I_{induced} = -\frac{(b-a)}{2\pi R}\frac{d}{dt}\int_{y=a}^{b}\frac{\mu_0 K t^2}{y}\,dy$$

$$= -\frac{(b-a)}{2\pi R}\ln\frac{b}{a}\mu_0 K\frac{d}{dt}t^2$$

$$= -\frac{2tK(b-a)}{2\pi R}\mu_0 \ln b/a$$



Does ~~current~~ flow? Circle one: Yes ~~No~~
$I_{loop} = -\frac{tK\mu_0(b-a)}{\pi R}\ln b/a$