SOLUTIONS : EEL 3472 TEST #1, Feb 11, 2009

Problem 1.

On this one page, describe succinctly in words and, if you wish, a few equations, the differences between a transient, time-domain solution to the transmission line equations and a phasor solution. Give a practical case for which one would need to use a time-domain solution and a case for which one would need to use a phasor solution.

The transient solution involves following in time and space a time-waveform, in our case a step function, as it reflects sequentially from each end of the line. After an ∞ number of bounces, the system settles to a dc value for the case of an input step function. Propagating waveforms are $f(t \pm \sqrt{L'C'}\, z)$, where f is any function.

+20

The phasor solution is a steady state solution at one frequency, that is, the one-frequency generator has been "on" forever. The phasor solution handles a lossy line easier than the time domain but only at one frequency at a time. All signals $\sim e^{j\omega t} e^{\pm \gamma z}$, $\gamma = \alpha + j\beta$

Practical cases

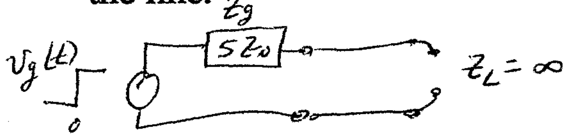
Phasor: The transmission line feeding a radar antenna - one frequency, modulated some - a transmission line carrying any narrow band signal

+10

Time domain: The transmission line carrying digital signals from one place on an integrated circuit to another; a transmission line carrying any wide-band signal

Problem 2.

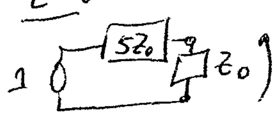
A unit step function is applied at $t = 0$ to a lossless transmission line of length ℓ whose characteristic impedance is Z_0 and whose Thevenin-equivalent generator impedance is $5Z_0$. The load impedance is an open circuit, $Z_L = \infty$. Plot the voltage and the current at $z = 3\ell/4$, that is, $\ell/4$ from the load, from $t = 0$ to $t = 3T$ where $T = \ell/c$ and "c" is the speed of light (the speed of the voltage and current on the line). Find the voltage and current at $3\ell/4$ after an infinite number of reflections have occurred from both ends of the line.



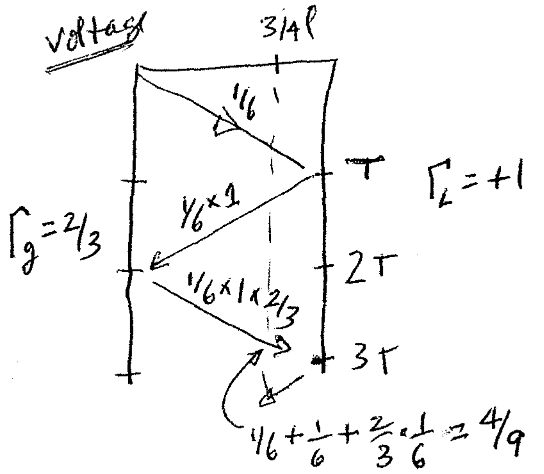
$\Gamma_{L\,volt} = \frac{\infty - Z_0}{\infty + Z_0} = +1$, $\Gamma_{L\,current} = -1$

$\Gamma_{g\,volt} = \frac{5Z_0 - Z_0}{5Z_0 + Z_0} = \frac{2}{3}$, $\Gamma_{g\,current} = -2/3$

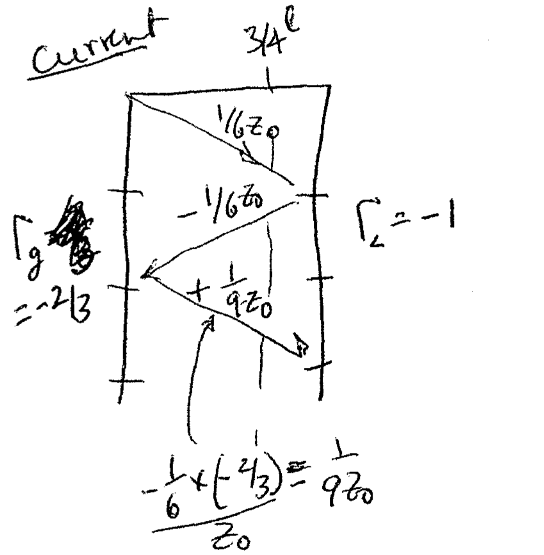
at $t = 0$



$V_1^+ = \frac{Z_0}{6Z_0} = \frac{1}{6}$, $I_1^+ = \frac{V_1^+}{Z_0} = \frac{1}{6Z_0}$



when current goes to zero after an ∞ number of reflections, each smaller than the previous, the voltage at the open circuit is $V_g = 1$ via dc voltage division $V_{LOAD} = \frac{\infty}{5Z_0 + \infty} =$ 1

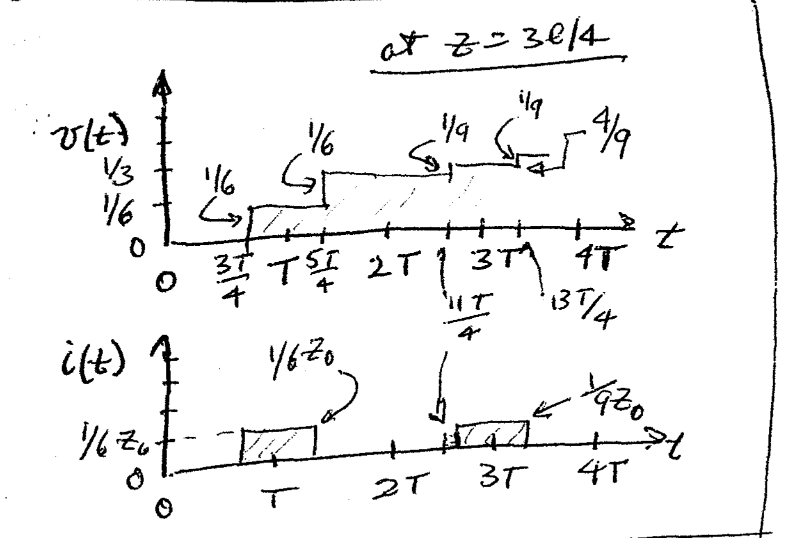


+10

+10

+5

+5



$V(t = \infty) = 1$

$I(t = \infty) = 0$

Problem 3.

A voltage generator with $v_g(t) = A\sin(2\pi 10^{10}t + \pi/4)$ and internal impedance 200 Ohms is connected to a lossless transmission line characterized by $L' = 10^{-6}\ H/m$ and $C' = 10^{-10}\ F/m$. If the load impedance is 100 Ohms and the line is 100 meters long, find:

$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{10^{-6}}{10^{-10}}} = 100\ \Omega$

(a) The phasor voltage source $\tilde{V}_g$

(b) The wave length λ on the line.

(c) The propagation constant β

$u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{10^{-16}}} = 10^8\ m/s$

(d) The voltage and current reflection coefficients at the load

(e) The input impedance Z_{in} to the line

(f) The phasor input voltage to the line $\tilde{V}_i$

(g) The SWR (Standing Wave Ratio) on the line

$\omega = 2\pi \times 10^{10}$ rad/s

$f = 10^{10}$ Hz

(a) $v_g = A\sin(2\pi\times 10^{10}t + \frac{\pi}{4}) = A\cos(2\pi\times 10^{10}t + \frac{\pi}{4} - \frac{\pi}{2})$

$\hat{V}_g = Ae^{-j\pi/4}$

(b) $\lambda = \frac{u_p}{f} = \frac{1/\sqrt{L'C'}}{10^{10}} = \frac{10^8}{10^{10}} = 10^{-2}\ m$

(c) $\beta = 2\pi/\lambda = \frac{2\pi}{10^{-2}} = 200\pi$

(d) $\Gamma_V = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-100}{100+100} = 0, \quad \Gamma_V = -\Gamma_I = 0$

(e) $Z_{in}(-l) = Z_0\left(\frac{\overset{100}{Z_L} + j\overset{100}{Z_0}\tan\beta l}{\underset{100}{Z_0} + j\underset{100}{Z_L}\tan\beta l}\right) = Z_0 = 100 = \sqrt{\frac{L'}{C'}}$

(f) $\tilde{V}_i = \frac{\hat{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{Ae^{-j\pi/4}\,100}{200+100}$

(g) $SWR = \frac{1 + \overset{\to 0}{|\Gamma|}}{1 - \underset{0}{|\Gamma_V|}} = 1$

+5	(a) $\hat{V}_g = Ae^{-j\pi/4}$
+4	(b) $\lambda = 10^{-2}\ m$
+4	(c) $\beta = 200\pi$
+4	(d) $\Gamma_{LV} = 0$; $\Gamma_{LI} = 0$
+5	(e) $Z_{in} = 100\ \Omega$
+4	(f) $\hat{V}_i = \frac{A}{3}e^{-j\pi/4}$ volts
+4	(g) SWR = 1