SOLUTIONS - HW # 6 - EEL 3472 - due March 2, 2009

1

Problem 4.3 Find the total charge contained in a cone defined by $R \leq 2$ m and $0 \leq \theta \leq \pi/4$, given that $\rho_v = 10R^2\cos^2\theta$ (mC/m^3).

Solution: For the cone of Fig. P4.3, application of Eq. (4.5) gives

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/4}\int_{R=0}^{2} \overbrace{10R^2\cos^2\theta}^{\rho_v}\, \overbrace{R^2\sin\theta\, dR\, d\theta\, d\phi}^{dV}$$

$$= \left(\frac{-2}{3}R^5\phi\cos^3\theta\right)\Bigg|_{R=0}^{2}\Bigg|_{\theta=0}^{\pi/4}\Bigg|_{\phi=0}^{2\pi}$$

$$= \frac{128\pi}{3}\left(1-\left(\frac{\sqrt{2}}{2}\right)^3\right) = 86.65 \text{ (mC)}.$$

$dQ = \rho_v dV$

$dV = R^2 \sin\theta\, d\theta\, d\phi\, dR$

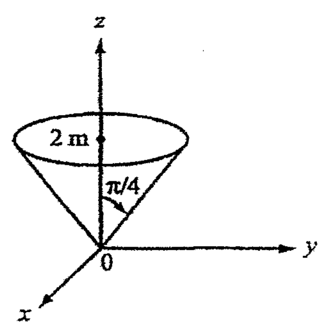


Figure P4.3: Cone of Problem 4.3.

not assigned but interesting

Problem 4.4 If the line charge density is given by $\rho_l = 24y^2$ (mC/m), find the total charge distributed on the y-axis from $y = -5$ to $y = 5$.

Solution:

$$Q = \int_{-5}^{5} \underbrace{\rho_l \overbrace{dy}^{dl}}_{dQ = \rho_l\, dl} = \int_{-5}^{5} 24y^2\, dy = \frac{24y^3}{3}\Bigg|_{-5}^{5} = 2000 \text{ mC} = 2 \text{ C}.$$

Problem 4.5 Find the total charge on a circular disk defined by $r \leq a$ and $z = 0$ if:
(a) $\rho_s = \rho_{s0} \cos\phi$ (C/m^2),
(b) $\rho_s = \rho_{s0} \sin^2\phi$ (C/m^2),
(c) $\rho_s = \rho_{s0} e^{-r}$ (C/m^2),
(d) $\rho_s = \rho_{s0} e^{-r} \sin^2\phi$ (C/m^2),
where ρ_{s0} is a constant.

Solution:
(a)

$$Q = \int \rho_s \, ds = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} \cos\phi \, \overbrace{r\,dr\,d\phi}^{ds} = \rho_{s0} \left. \frac{r^2}{2} \right|_0^a \left. \sin\phi \right|_0^{2\pi} = 0.$$

(b)

$$Q = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} \sin^2\phi \, \overbrace{r\,dr\,d\phi}^{ds} = \rho_{s0} \left. \frac{r^2}{2} \right|_0^a \int_0^{2\pi} \left(\frac{1-\cos 2\phi}{2} \right) d\phi$$
$$= \frac{\rho_{s0} a^2}{4} \left(\phi - \frac{\sin 2\phi}{2} \right) \Bigg|_0^{2\pi} = \frac{\pi a^2}{2} \rho_{s0}.$$

(c)

$$Q = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} r\,dr\,d\phi = 2\pi\rho_{s0} \int_0^a r e^{-r}\,dr$$
$$= 2\pi\rho_{s0} \left[-re^{-r} - e^{-r} \right]_0^a$$
$$= 2\pi\rho_{s0} [1 - e^{-a}(1+a)].$$

(d)

$$Q = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} \sin^2\phi \; r\,dr\,d\phi$$
$$= \rho_{s0} \int_{r=0}^{a} re^{-r}\,dr \int_{\phi=0}^{2\pi} \sin^2\phi \, d\phi$$
$$= \rho_{s0}[1 - e^{-a}(1+a)] \cdot \pi = \pi\rho_{s0}[1 - e^{-a}(1+a)].$$

Problem 4.7 If $\mathbf{J} = \hat{\mathbf{R}} 5/R$ (A/m^2), find I through the surface $R = 5$ m.

Solution: Using Eq. (4.12), we have

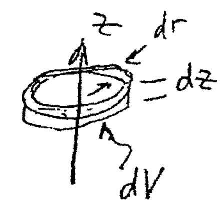
$\vec{ds} = R^2 \sin\theta \, d\theta \, d\phi \, \hat{R}$

$$I = \int_S \mathbf{J} \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \overbrace{\left(\hat{\mathbf{R}} \frac{5}{R} \right) \cdot (\hat{\mathbf{R}} R^2 \sin\theta \, d\theta \, d\phi)}^{\vec{J} \cdot \vec{ds}}$$
$$= -5R\phi\cos\theta \Bigg|_{R=5} \Bigg|_{\theta=0}^{\pi} \Bigg|_{\phi=0}^{2\pi} = 100\pi = 314.2 \quad \text{(A)}.$$

(3)

Problem 4.8 An electron beam shaped like a circular cylinder of radius r_0 carries a charge density given by

$$\rho_v = \left(\frac{-\rho_0}{1+r^2}\right) \quad (\text{C/m}^3),$$



where ρ_0 is a positive constant and the beam's axis is coincident with the z-axis.

(a) Determine the total charge contained in length L of the beam.

(b) If the electrons are moving in the $+z$-direction with uniform speed u, determine the magnitude and direction of the current crossing the z-plane.

Solution:

(a)

dV , rdrdφ dz with the dφ already integrated 0 → 2π

$$Q = \int_{r=0}^{r_0}\int_{z=0}^{L} \rho_v \, d\mathcal{V} = \int_{r=0}^{r_0}\int_{z=0}^{L} \left(\frac{-\rho_0}{1+r^2}\right) \underbrace{2\pi r \, dr \, dz}_{dV}$$
$$= -2\pi\rho_0 L \int_0^{r_0} \frac{r}{1+r^2}\, dr = -\pi\rho_0 L \ln(1+r_0^2).$$

(b)

$$\mathbf{J} = \rho_v \mathbf{u} = -\hat{\mathbf{z}}\frac{u\rho_0}{1+r^2} \quad (\text{A/m}^2),$$
$$I = \int \mathbf{J}\cdot d\mathbf{s}$$
$$= \int_{r=0}^{r_0}\int_{\phi=0}^{2\pi} \left(-\hat{\mathbf{z}}\frac{u\rho_0}{1+r^2}\right)\cdot \hat{\mathbf{z}} r\, dr\, d\phi$$
$$= -2\pi u\rho_0 \int_0^{r_0} \frac{r}{1+r^2}\, dr = -\pi u\rho_0 \ln(1+r_0^2) \quad (\text{A}).$$

Current direction is along $-\hat{\mathbf{z}}$.

4

Problem 4.9 A square with sides 2 m each has a charge of 40 μC at each of its four corners. Determine the electric field at a point 5 m above the center of the square.

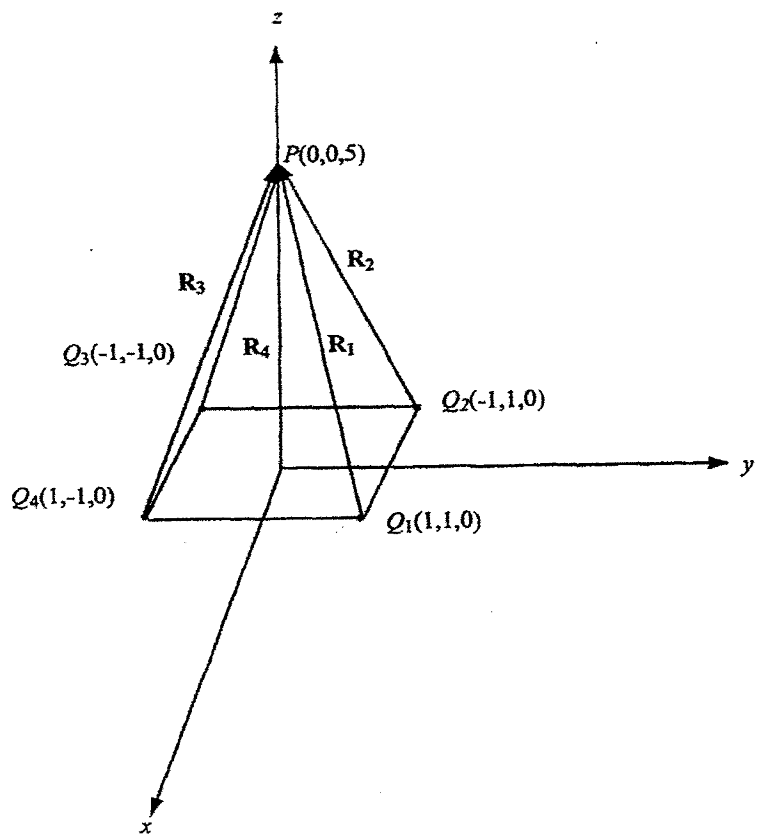


Figure P4.9: Square with charges at the corners.

Solution: The distance $|R|$ between any of the charges and point P is

$$|R| = \sqrt{1^2+1^2+5^2} = \sqrt{27}.$$

$$\begin{aligned}
\mathbf{E} &= \frac{Q}{4\pi\varepsilon_0}\left[\frac{\mathbf{R}_1}{|\mathbf{R}|^3}+\frac{\mathbf{R}_2}{|\mathbf{R}|^3}+\frac{\mathbf{R}_3}{|\mathbf{R}|^3}+\frac{\mathbf{R}_4}{|\mathbf{R}|^3}\right] \\
&= \frac{Q}{4\pi\varepsilon_0}\left[\frac{-\hat{\mathbf{x}}-\hat{\mathbf{y}}+\hat{\mathbf{z}}5}{(27)^{3/2}}+\frac{\hat{\mathbf{x}}-\hat{\mathbf{y}}+\hat{\mathbf{z}}5}{(27)^{3/2}}+\frac{-\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}5}{(27)^{3/2}}+\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}5}{(27)^{3/2}}\right] \\
&= \hat{\mathbf{z}}\frac{5Q}{(27)^{3/2}\pi\varepsilon_0} = \hat{\mathbf{z}}\frac{5\times 40\ \mu\text{C}}{(27)^{3/2}\pi\varepsilon_0} = \frac{1.42}{\pi\varepsilon_0}\times 10^{-6}\ \text{(V/m)} = \hat{\mathbf{z}}51.2\ \text{(kV/m)}.
\end{aligned}$$

Problem 4.12 A line of charge with uniform density $\rho_l = 8$ (μC/m) exists in air along the z-axis between $z = 0$ and $z = 5$ cm. Find **E** at (0,10 cm,0).

Solution: Use of Eq. (4.21c) for the line of charge shown in Fig. P4.12 gives

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int_{l'} \hat{\mathbf{R}}' \frac{\rho_l \, dl'}{R'^2},$$

$$R' = \hat{\mathbf{y}}0.1 - \hat{\mathbf{z}}z$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{z=0}^{0.05} (8\times 10^{-6}) \frac{(\hat{\mathbf{y}}0.1 - \hat{\mathbf{z}}z)}{[(0.1)^2 + z^2]^{3/2}}\, dz$$

$$= \frac{8\times 10^{-6}}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{y}}10z + \hat{\mathbf{z}}}{\sqrt{(0.1)^2 + z^2}}\right]\Bigg|_{z=0}^{0.05}$$

$$= 71.86\times 10^3\,[\hat{\mathbf{y}}4.47 - \hat{\mathbf{z}}1.06] = \hat{\mathbf{y}}321.4\times 10^3 - \hat{\mathbf{z}}76.2\times 10^3 \quad \text{(V/m)}.$$

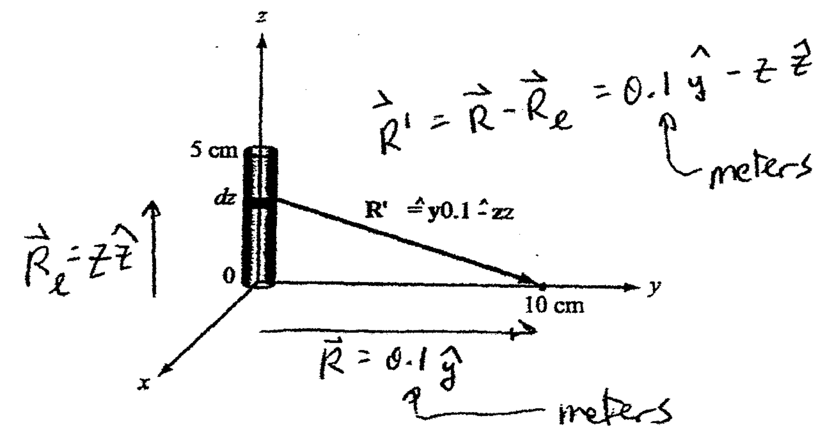


Figure P4.12: Line charge.

6

Problem 4.15 Repeat Example 4-5 for the circular disk of charge of radius a, but in the present case assume the surface charge density to vary with r as

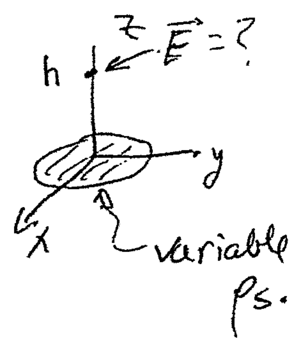
$$\rho_s = \rho_{s0} r^2 \quad (\text{C/m}^2),$$

where ρ_{s0} is a constant.

Solution: We start with the expression for $d\mathbf{E}$ given in Example 4-5 but we replace ρ_s with $\rho_{s0} r^2$:

$$d\mathbf{E} = \hat{\mathbf{z}} \frac{h}{4\pi\varepsilon_0 (r^2 + h^2)^{3/2}} (2\pi \rho_{s0} r^3 \, dr),$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_0^a \frac{r^3 \, dr}{(r^2 + h^2)^{3/2}}.$$



To perform the integration, we use

$$R^2 = r^2 + h^2,$$

$$2R\,dR = 2r\,dr,$$

$$\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \int_h^{(a^2+h^2)^{1/2}} \frac{(R^2 - h^2)\, dR}{R^2}$$

$$= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\int_h^{(a^2+h^2)^{1/2}} dR - \int_h^{(a^2+h^2)^{1/2}} \frac{h^2}{R^2}\, dR \right]$$

$$= \hat{\mathbf{z}} \frac{\rho_{s0} h}{2\varepsilon_0} \left[\sqrt{a^2 + h^2} + \frac{h^2}{\sqrt{a^2 + h^2}} - 2h \right].$$

Problem 4.20 Given the electric flux density

$$\mathbf{D} = \hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y) \quad (\text{C/m}^2),$$

determine

(a) ρ_v by applying Eq. (4.26),

(b) the total charge Q enclosed in a cube 2 m on a side, located in the first octant with three of its sides coincident with the x-, y-, and z-axes and one of its corners at the origin, and

(c) the total charge Q in the cube, obtained by applying Eq. (4.29).

Solution:

(a) By applying Eq. (4.26)

$$\rho_v = \nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(2x+2y) + \frac{\partial}{\partial y}(3x-2y) = 0.$$

(b) Integrate the charge density over the volume as in Eq. (4.27):

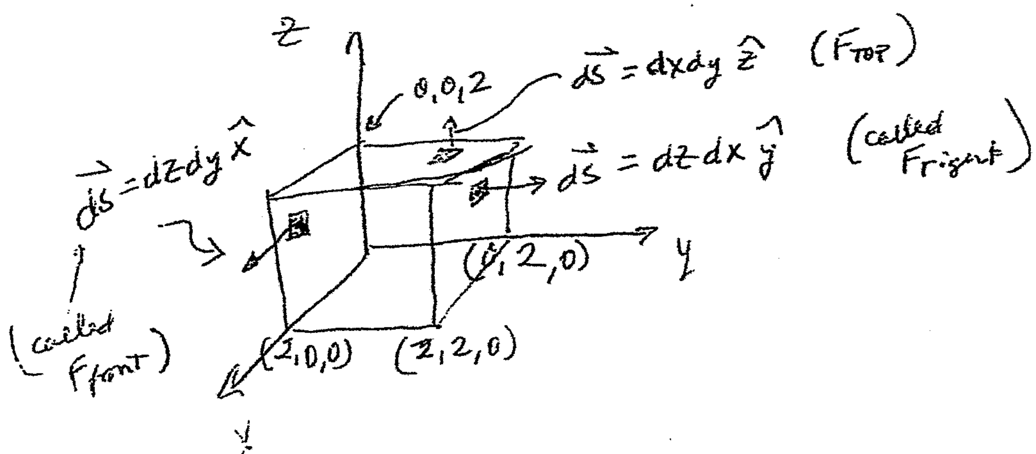
$$Q = \int_{\mathcal{V}} \nabla \cdot \mathbf{D}\, d\mathcal{V} = \int_{x=0}^{2}\int_{y=0}^{2}\int_{z=0}^{2} 0\, dx\, dy\, dz = 0.$$

(c) Apply Gauss' law to calculate the total charge from Eq. (4.29)

$$Q = \oint \mathbf{D} \cdot d\mathbf{s} = F_{\text{front}} + F_{\text{back}} + F_{\text{right}} + F_{\text{left}} + F_{\text{top}} + F_{\text{bottom}},$$

$$F_{\text{front}} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{x=2} \cdot (\hat{\mathbf{x}}\, dz\, dy)$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\Big|_{x=2}\, dz\, dy = \left(2z\left(2y + \frac{1}{2}y^2\right)\Big|_{z=0}^{2}\right)\Bigg|_{y=0}^{2} = 24,$$



$$F_{\text{back}} = \int_{y=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y))\bigg|_{x=0} \cdot (-\hat{\mathbf{x}}\,dz\,dy)$$

$$= -\int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\bigg|_{x=0} dz\,dy = -\left(zy^2\bigg|_{z=0}^{2}\right)\bigg|_{y=0}^{2} = -8,$$

$$F_{\text{right}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y))\bigg|_{y=2} \cdot (\hat{\mathbf{y}}\,dz\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} (3x-2y)\bigg|_{y=2} dz\,dx = \left(z\left(\frac{3}{2}x^2 - 4x\right)\bigg|_{z=0}^{2}\right)\bigg|_{x=0}^{2} = -4,$$

$$F_{\text{left}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y))\bigg|_{y=0} \cdot (-\hat{\mathbf{y}}\,dz\,dx)$$

$$= -\int_{x=0}^{2}\int_{z=0}^{2} (3x-2y)\bigg|_{y=0} dz\,dx = -\left(z\left(\frac{3}{2}x^2\right)\bigg|_{z=0}^{2}\right)\bigg|_{x=0}^{2} = -12,$$

$$F_{\text{top}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y))\bigg|_{z=2} \cdot (\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} 0\bigg|_{z=2} dy\,dx = 0,$$

$$F_{\text{bottom}} = \int_{x=0}^{2}\int_{z=0}^{2} (\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y))\bigg|_{z=0} \cdot (\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2} 0\bigg|_{z=0} dy\,dx = 0.$$

Thus $Q = \oint \mathbf{D} \cdot d\mathbf{s} = 24 - 8 - 4 - 12 + 0 + 0 = 0.$

Problem 4.21 Repeat Problem 4.20 for $\mathbf{D} = \hat{\mathbf{x}}xy^3z^3$ (C/m^2).

Solution:

(a) From Eq. (4.26), $\rho_v = \nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(xy^3z^3) = y^3z^3.$

(b) Total charge Q is given by Eq. (4.27):

$$Q = \int_{\mathcal{V}} \nabla \cdot \mathbf{D}\,d\mathcal{V} = \int_{z=0}^{2}\int_{y=0}^{2}\int_{x=0}^{2} y^3z^3\,dx\,dy\,dz = \frac{xy^4z^4}{16}\bigg|_{x=0}^{2}\bigg|_{y=0}^{2}\bigg|_{z=0}^{2} = 32\text{ C}.$$

(c) Using Gauss' law we have

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = F_{\text{front}} + F_{\text{back}} + F_{\text{right}} + F_{\text{left}} + F_{\text{top}} + F_{\text{bottom}}.$$

Note that $\mathbf{D} = \hat{\mathbf{x}} D_x$, so only F_{front} and F_{back} (integration over $\hat{\mathbf{z}}$ surfaces) will contribute to the integral.

$$F_{\text{front}} = \int_{z=0}^{2} \int_{y=0}^{2} (\hat{\mathbf{x}} xy^3 z^3)\Big|_{x=2} \cdot (\hat{\mathbf{x}}\, dy\, dz)$$

$$= \int_{z=0}^{2} \int_{y=0}^{2} xy^3 z^3\Big|_{x=2} dy\, dz = \left(2\left(\frac{y^4 z^4}{16}\right)\Big|_{y=0}^{2} \right)\Big|_{z=0}^{2} = 32,$$

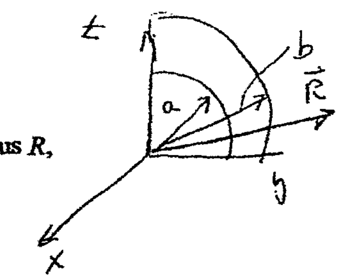
$$F_{\text{back}} = \int_{z=0}^{2} \int_{y=0}^{2} (\hat{\mathbf{x}} xy^3 z^3)\Big|_{x=0} \cdot (-\hat{\mathbf{x}}\, dy\, dz) = -\int_{z=0}^{2} \int_{y=0}^{2} xy^3 z^3\Big|_{x=0} dy\, dz = 0.$$

Thus $Q = \oint \mathbf{D} \cdot d\mathbf{s} = 32 + 0 + 0 + 0 + 0 + 0 = 32$ C.

Problem 4.22 Charge Q_1 is uniformly distributed over a thin spherical shell of radius a, and charge Q_2 is uniformly distributed over a second spherical shell of radius b, with $b > a$. Apply Gauss's law to find $\mathbf{E}$ in the regions $R < a$, $a < R < b$, and $R > b$.

Solution: Using symmetry considerations, we know $\mathbf{D} = \hat{\mathbf{R}} D_R$. From Table 3.1, $d\mathbf{s} = \hat{\mathbf{R}} R^2 \sin\theta\, d\theta\, d\phi$ for an element of a spherical surface. Using Gauss's law in integral form (Eq. (4.29)),

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{\text{tot}},$$



where Q_{tot} is the total charge enclosed in S. For a spherical surface of radius R,

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} (\hat{\mathbf{R}} D_R) \cdot (\hat{\mathbf{R}} R^2 \sin\theta\, d\theta\, d\phi) = Q_{\text{tot}},$$

$$D_R R^2 (2\pi) [-\cos\theta]_0^\pi = Q_{\text{tot}},$$

$$D_R = \frac{Q_{\text{tot}}}{4\pi R^2}.$$

From Eq. (4.15), we know a linear, isotropic material has the constitutive relationship $\mathbf{D} = \varepsilon \mathbf{E}$. Thus, we find $\mathbf{E}$ from $\mathbf{D}$.

(a) In the region $R < a$,

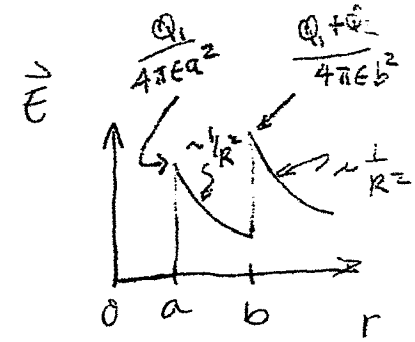
$$Q_{\text{tot}} = 0, \qquad \mathbf{E} = \hat{\mathbf{R}}E_R = \frac{\hat{\mathbf{R}}Q_{\text{tot}}}{4\pi R^2 \varepsilon} = 0 \quad \text{(V/m)}.$$

(b) In the region $a < R < b$,

$$Q_{\text{tot}} = Q_1, \qquad \mathbf{E} = \hat{\mathbf{R}}E_R = \frac{\hat{\mathbf{R}}Q_1}{4\pi R^2 \varepsilon} \quad \text{(V/m)}.$$

(c) In the region $R > b$,

$$Q_{\text{tot}} = Q_1 + Q_2, \qquad \mathbf{E} = \hat{\mathbf{R}}E_R = \frac{\hat{\mathbf{R}}(Q_1 + Q_2)}{4\pi R^2 \varepsilon} \quad \text{(V/m)}.$$



Problem 4.23 The electric flux density inside a dielectric sphere of radius a centered at the origin is given by

$$\mathbf{D} = \hat{\mathbf{R}}\rho_0 R \quad \text{(C/m}^2\text{)},$$

where ρ_0 is a constant. Find the total charge inside the sphere.

Solution:

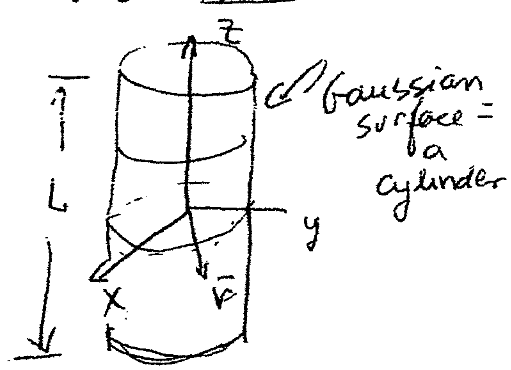
$$Q = \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \hat{\mathbf{R}}\rho_0 R \cdot \overbrace{\hat{\mathbf{R}}R^2 \sin\theta \, d\theta \, d\phi}^{\vec{ds}} \Big|_{R=a}$$
$$= 2\pi\rho_0 a^3 \int_0^{\pi} \sin\theta \, d\theta = -2\pi\rho_0 a^3 \cos\theta \big|_0^{\pi} = 4\pi\rho_0 a^3 \quad \text{(C)}.$$

Problem 4.24 In a certain region of space, the charge density is given in cylindrical coordinates by the function:

$$\rho_v = 50 r e^{-r} \quad \text{(C/m}^3\text{)}.$$

Apply Gauss's law to find **D**.

Solution:



11

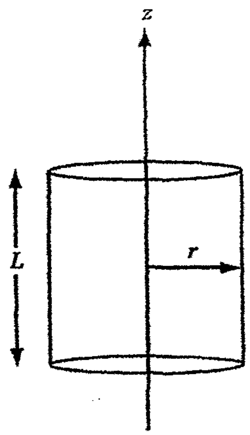


Figure P4.24: Gaussian surface.

Method 1: Integral Form of Gauss's Law

Since ρ_v varies as a function of r only, so will $\mathbf{D}$. Hence, we construct a cylinder of radius r and length L, coincident with the z-axis. Symmetry suggests that $\mathbf{D}$ has the functional form $\mathbf{D} = \hat{\mathbf{r}} D$. Hence,

draw picture to prove

over Gaussian cylinder at radius r

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q,$$

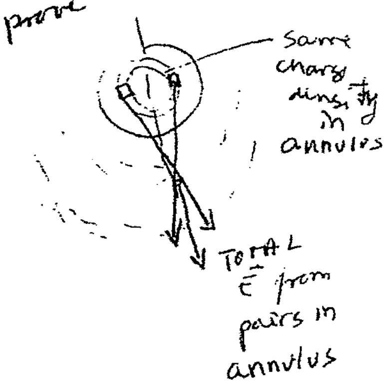
all charge inside r

$$\int \hat{\mathbf{r}} D \cdot d\mathbf{s} = D(2\pi r L),$$

$$Q = 2\pi L \int_0^r 50 r e^{-r} \cdot r \, dr$$

$$= 100\pi L[-r^2 e^{-r} + 2(1 - e^{-r}(1+r))],$$

$$\mathbf{D} = \hat{\mathbf{r}} D = \hat{\mathbf{r}} 50 \left[\frac{2}{r}(1 - e^{-r}(1+r)) - r e^{-r} \right].$$



Method 2: Differential Method

$$\nabla \cdot \mathbf{D} = \rho_v, \qquad \mathbf{D} = \hat{\mathbf{r}} D_r,$$

with D_r being a function of r.

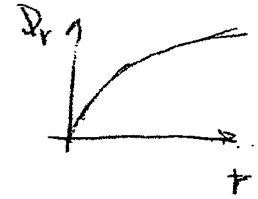
$$\frac{1}{r} \frac{\partial}{\partial r}(r D_r) = 50 r e^{-r},$$

$$\frac{\partial}{\partial r}(rD_r) = 50r^2e^{-r},$$

$$\int_0^r \frac{\partial}{\partial r}(rD_r)\,dr = \int_0^r 50r^2e^{-r}\,dr,$$

$$rD_r = 50[2(1-e^{-r}(1+r)) - r^2e^{-r}],$$

$$\mathbf{D} = \hat{\mathbf{r}}rD_r = \hat{\mathbf{r}}50\left[\frac{2}{r}(1-e^{-r}(1+r)) - re^{-r}\right].$$



Problem 4.25 An infinitely long cylindrical shell extending between $r = 1$ m and $r = 3$ m contains a uniform charge density ρ_{v0}. Apply Gauss's law to find $\mathbf{D}$ in all regions.

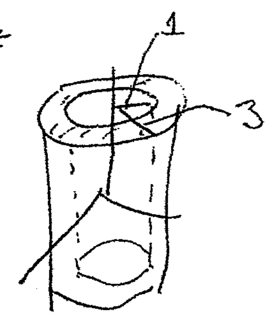
Solution: For $r < 1$ m, $\mathbf{D} = 0$.
For $1 \le r \le 3$ m,

$$\int_{z=0}^{L}\int_{r'=1}^{r} \rho_{v0}\,\overbrace{r'\,d\phi\,dz}^{dV}$$

$$\oint_S \hat{\mathbf{r}}D_r \cdot d\mathbf{s} = Q,$$

$$D_r \cdot 2\pi rL = \rho_{v0}\cdot\pi L(r^2 - 1^2),$$

$$\mathbf{D} = \hat{\mathbf{r}}D_r = \hat{\mathbf{r}}\frac{\rho_{v0}\pi L(r^2-1)}{2\pi rL} = \hat{\mathbf{r}}\frac{\rho_{v0}(r^2-1)}{2r}, \qquad 1 \le r \le 3 \text{ m}.$$



For $r \ge 3$ m,

$$\int_{z=0}^{L}\int_{r=1}^{2} \rho_{v0}\, r\,d\phi\,dz$$

$$D_r \cdot 2\pi rL = \rho_{v0}\pi L(3^2 - 1^2) = 8\rho_{v0}\pi L,$$

$$\mathbf{D} = \hat{\mathbf{r}}D_r = \hat{\mathbf{r}}\frac{4\rho_{v0}}{r}, \qquad r \ge 3 \text{ m}.$$