EEL 3412 HW #A SOLUTIONS – FEB 6, 2004 (1)
(4 problems / 4 pages)

#1

Problem 2.21 A voltage generator with $v_g(t) = 5\cos(2\pi \times 10^9 t)$ V and internal impedance $Z_g = 50\ \Omega$ is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and it is terminated in a load with impedance $Z_L = (100 - j100)\ \Omega$. Find

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) the input voltage $\widetilde{V}_i$ and input current $\widetilde{I}_i$.

Solution:

(a) From Eq. (2.49a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.63),

$$Z_{in}(-\ell) = Z_0\left[\frac{1 + \Gamma_v e^{-j\beta\ell}}{1 - \Gamma_v e^{-j\beta\ell}}\right]$$

$$\begin{aligned}
Z_{in} &= Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right) \\
&= 50\left(\frac{(100 - j100) + j50\tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}{50 + j(100 - j100)\tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}\right) \\
&= 50\left(\frac{(100 - j100) + j50\tan\left(\frac{\pi}{3} \text{ rad}\right)}{50 + j(100 - j100)\tan\left(\frac{\pi}{3} \text{ rad}\right)}\right) = (12.5 - j12.7)\ \Omega.
\end{aligned}$$

An alternative solution to this part involves the solution to part (a) and Eq. (2.61).

(c) In phasor domain, $\widetilde{V}_g = 5\,\text{V}\,e^{j0^\circ}$. From Eq. (2.64), $v_g(t) = \mathrm{Re}\left[\widetilde{V}_g e^{j\omega t}\right] = 5\cos\omega t$

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \quad \text{(V)},$$

and also from Eq. (2.64),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j11.5^\circ} \quad \text{(mA)}.$$

#3

$S = \frac{1+|\Gamma|}{1-|\Gamma|}$, Γ = voltage reflection coef at load

$S(1-|\Gamma|) = 1+|\Gamma|$ $\qquad S = \frac{15}{5} = 3$

$S - S|\Gamma| = 1+|\Gamma|$

$S-1 = S|\Gamma| + |\Gamma| = |\Gamma|(1+S)$

$|\Gamma| = \frac{S-1}{S+1} = \frac{15/5-1}{15/5+1} = \frac{3-1}{3+1} = \frac{2}{4} = 1/2$

various possibilities

$\Gamma = 1/2, -1/2$ for a real load $Z_L = R_L$

$|\Gamma| = 1/2$

for a complex or imaginary load

$\Gamma = \pm\frac{1}{2}e^{\pm j\theta_r}$, $|\Gamma| = 1/2$

for pure imaginary load

$\theta_r = 90°$ (? see below)

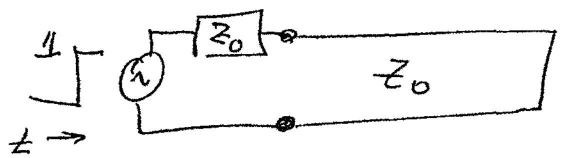
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$, Z_0 is real

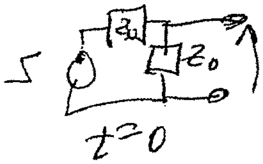
you can calculate what Z_L has to be in the cases above – if you do that, you will see that a pure imaginary load is inconsistent with $|\Gamma| = \frac{1}{2}$; if $Z_L = jX$

$|\Gamma| = \left|\frac{+jX - Z_0}{+jX + Z_0}\right| = 1$, so $\theta_r \neq 90°$ for a solution

is not possible

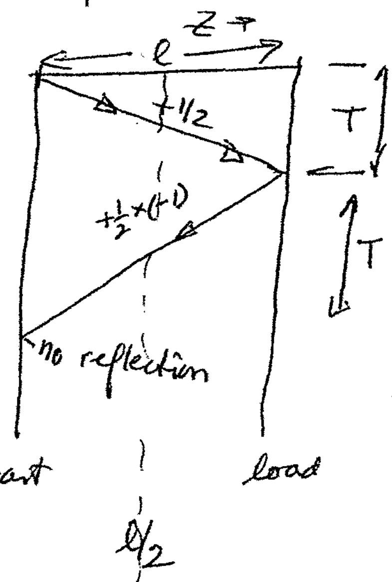
(3)





$V_i = 1 \cdot \frac{Z_0}{2Z_0} = \frac{1}{2}$

for voltage



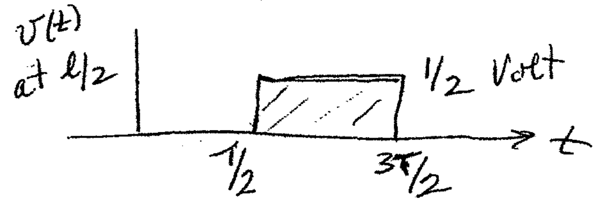
$\Gamma_{gen} = 0 \quad \left(\frac{Z_0 - Z_0}{Z_0 + Z_0}\right)$

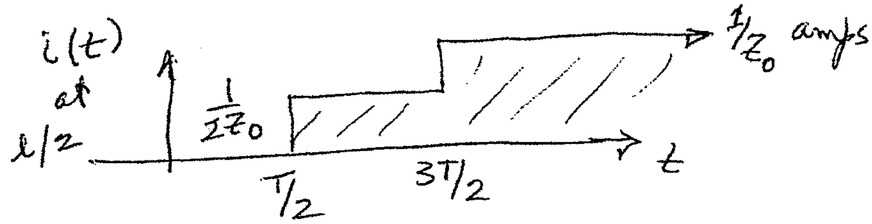
(4)

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ (Z_L = 0)

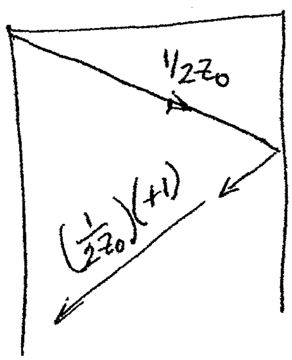
$\Gamma_L = -1$ (voltage)

$\Gamma_{gen} = 0$ (voltage)





current

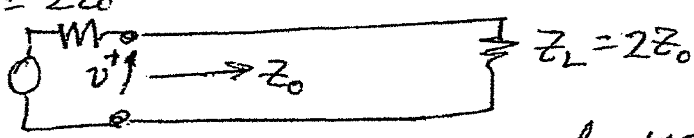


after 2T

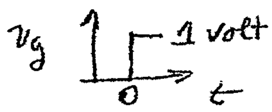
$V(l/2) = 0$

Solution to ~~4~~ 4th problem

4



lossless line, Z_0, $u_p = c$

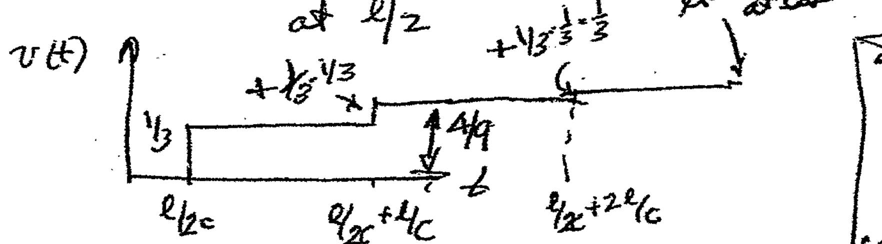


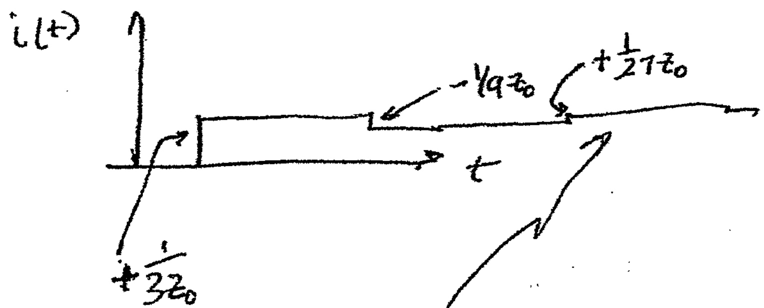
$$v^+ = \frac{\overset{1\,\text{volt}}{v_g}\, Z_0}{2Z_0 + Z_0} = 1/3 \text{ volt}$$

$$\Gamma_{L\,\text{volt}} = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{Z_0}{3Z_0} = +\frac{1}{3}\,, \quad \Gamma_{L\,\text{current}} = -1/3$$

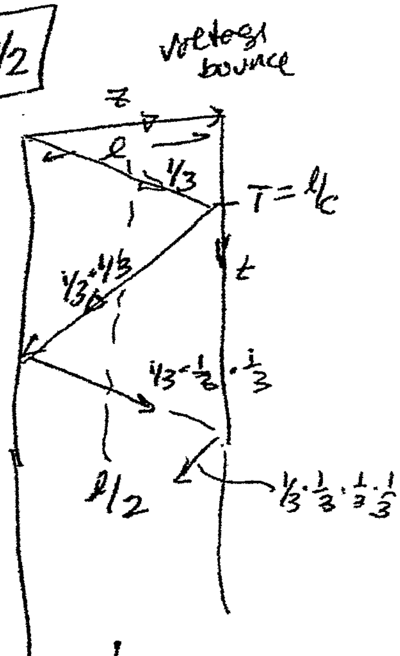
limit is $\frac{1}{2}$ at ∞

at $\ell/2$





current reflections oscillate and settle at $i(t) = \frac{1}{4Z_0}$



eventually $v(t)$ everywhere on the line is $\frac{1}{2}$, each step being $1/3$ of the previous.

At ∞, $v_g = 1/2$

$i(t) = \frac{1}{4Z_0}$