EEL 3472 HW #3 SOLUTIONS - JAN. 30, 2009

1.

Problem 2.10 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

Solution: From the definition of the Standing Wave Ratio given by Eq. (2.59),

$$S=\frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}}=\frac{1.5}{0.6}=2.5. = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Solving for the magnitude of the reflection coefficient in terms of S, as in Example 2-4,

$$|\Gamma|=\frac{S-1}{S+1}=\frac{2.5-1}{2.5+1}=0.43.$$

$$\left\{\begin{aligned} &S=\frac{1+|\Gamma|}{1-|\Gamma|} \\ &S-S|\Gamma| = 1+|\Gamma| \\ &S-1 = |\Gamma|+S|\Gamma| \\ &\frac{S-1}{S+1} = |\Gamma| \end{aligned}\right.$$

2.

Problem 2.12 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L=(30-j50)$ Ω. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,

(b) the standing-wave ratio on the line,

(c) the position of the voltage maximum nearest the load,

(d) the position of the current maximum nearest the load.

Solution:

(a) From Eq. (2.49a),

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}=\frac{(30-j50)-50}{(30-j50)+50}=0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.59),

$$S=\frac{1+|\Gamma|}{1-|\Gamma|}=\frac{1+0.57}{1-0.57}=3.65.$$

(c) From Eq. (2.56)

$$l_{\max}=\frac{\theta_r\lambda}{4\pi}+\frac{n\lambda}{2}=\frac{-79.8^\circ\times 8\text{ cm}}{4\pi}\frac{\pi\text{ rad}}{180^\circ}+\frac{n\times 8\text{ cm}}{2}$$
$$=-0.89\text{ cm}+4.0\text{ cm}=3.11\text{ cm}.$$

$n=1,2\cdots$ if $\theta_r<0$ ($\theta_r=-79.8^\circ$)

$n=0,1,2\cdots$ if $\theta_r>0$ which it is not

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.58),

$$l_{\min}=l_{\max}-\lambda/4=3.11\text{ cm}-8\text{ cm}/4=1.11\text{ cm}.$$

Problem 3.

A voltage generator with $\upsilon_g(t) = A\sin(10^{10}t)$ and internal impedance 10 Ohms is connected to a lossless transmission line characterized by $L' = 10^{-6}\,H/m$ and $C' = 10^{-10}\,F/m$. If the load impedance is 100 Ohms and the line is 10 meters long, find:

(a) The phasor voltage source $\tilde{V}_g$
(b) The wave length λ on the line
(c) The reflection coefficient at the load
(d) The input impedance Z_{in} to the line
(e) The phasor input voltage to the line $\tilde{V}_i$

$$Z_o = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{10^{-6}}{10^{-10}}} = 100\ \Omega$$

$$u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{10^{-6}\cdot 10^{-10}}} = 10^{8}\ m/s \quad (1/3\,c)$$

$$\omega = 10^{10}\ rad/s \qquad f = \frac{10^{10}}{2\pi}$$

(a) $\upsilon_g = A\cos(10^{10}t - \pi/2)$

$$\tilde{V}_g = Ae^{-j\pi/2}$$

(b) $$\lambda = \frac{u_p}{f} = \frac{10^8}{10^{10}/2\pi} = 2\pi\times 10^{-2}\ m$$

(c) $$\Gamma_L = \frac{100-100}{100+100} = 0$$

(d) since line is matched at load, $Z_L = Z_0$, $Z_{in} = 100\ \Omega$,

or, $$Z_{in} = 100\,\underbrace{\left[\frac{100 + j\,100\tan\beta l}{100 + j\,100\tan\beta l}\right]}_{1}$$

(e) $$\tilde{V}_i = \frac{\overset{100}{Z_{in}}}{Z_{in} + \underset{10}{Z_g}}\,\tilde{V}_g$$

Answers

(a)	$\tilde{V}_g = Ae^{-j\pi/2}$
(b)	$\lambda = 2\pi\times 10^{-2}$ meters $(\pi/50)$
(c)	$\Gamma_L = 0$
(d)	$Z_{in} = 100\ \Omega$
(e)	$\tilde{V}_i = \frac{10}{11}Ae^{-j\pi/2}$