EEL 3472 - HW #2 SOLUTIONS - DUE FRIDAY JAN. 23, 2009

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Q2.1 – A transmission line is any structure or media that serves to transfer energy or information between two points.
– Transmission line effects should be considered when the wavelength of the signal is close to the length of the line ($l/\lambda \geq 0.01$).

Q2.2 – In a dispersive transmission line, the wave speed is a function of the frequency of the wave. The wave speed in a non-dispersive transmission line does not depend on the frequency of the wave. Multimodal signals will decompose and become distorted as they propagate in dispersive transmission lines.

Q2.3 - A TEM transmission line is one in which both the electric and magnetic fields are perpendicular (transverse) to the direction of propagation. They usually consist of two parallel conducting surfaces.

Q2.4 - The lumped-element circuit model serves to represent the physical processes in transmission lines as circuit elements, which are then subject to standard circuit analysis procedures.

R' accounts for the combined resistance per unit length of the two conductors.
L' accounts for the inductance of the conductors per unit length.
G' accounts for possible current flow through the dielectric between the conductors per unit length.
C' accounts for the capacitance between the two conductors per unit length.

σ_c and μ_c are the conductivity and permeability of the conductors, and σ, ε, and μ are the conductivity, permittivity and permeability of the dielectric.

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Problem 2.1 A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations is it reasonable to ignore the presence of the transmission line in the solution of the circuit:

(a) $l = 20$ cm, $f = 20$ kHz,
(b) $l = 50$ km, $f = 60$ Hz,
(c) $l = 20$ cm, $f = 600$ MHz,
(d) $l = 1$ mm, $f = 100$ GHz.

Solution: A transmission line is negligible when $l/\lambda \leq 0.01$.

(a) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(20\times10^{-2}\text{ m})\times(20\times10^{3}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 1.33\times10^{-5}$ (negligible).

(b) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(50\times10^{3}\text{ m})\times(60\times10^{0}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.01$ (borderline).

(c) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(20\times10^{-2}\text{ m})\times(600\times10^{6}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.40$ (nonnegligible).

(d) $\dfrac{l}{\lambda} = \dfrac{lf}{u_p} = \dfrac{(1\times10^{-3}\text{ m})\times(100\times10^{9}\text{ Hz})}{3\times10^{8}\text{ m/s}} = 0.33$ (nonnegligible).

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$$-\frac{\partial v(z,t)}{\partial z} = \cancel{R'}^{\,0} i(z,t) + L'\frac{\partial i(z,t)}{\partial t} \quad (a)$$

lossless

$R' = G' = 0$

$$-\frac{\partial i(z,t)}{\partial z} = \cancel{G'}^{\,0} v(z,t) + C'\frac{\partial v(z,t)}{\partial t} \quad (b)$$

take $\frac{\partial}{\partial z}$ of (a) and $\frac{\partial}{\partial t}$ of (b)

$$-\frac{\partial^2 v}{\partial z^2} = L'\frac{\partial}{\partial z}\frac{\partial}{\partial t} i(z,t) \quad (a')$$

$$-\frac{\partial}{\partial t}\frac{\partial}{\partial z} i(z,t) = C'\frac{\partial^2 v(z,t)}{\partial t^2} \quad (b')$$

order of $\frac{\partial}{\partial t}\frac{\partial}{\partial z}$ doesn't matter, $\frac{\partial}{\partial t}\frac{\partial}{\partial z} = \frac{\partial}{\partial z}\frac{\partial}{\partial t}$

substitute (b') in (a')

$$-\frac{\partial^2 v}{\partial z^2} = -L'C'\frac{\partial^2 v}{\partial t^2}$$

$$\boxed{\frac{\partial^2 v(z,t)}{\partial z^2} - L'C'\frac{\partial^2 v(z,t)}{\partial t^2} = 0}$$

wave equation for $v(z,t)$

show that $A\left[\cos(\omega t - \omega\sqrt{L'C'}z - \phi_0)\right]$ is one solution (of many)

$$\frac{\partial v}{\partial z} = -A\left[\sin(\quad\right](-\omega\sqrt{L'C'})$$

$$\frac{\partial^2 v}{\partial z^2} = -A\left[\cos(\quad\right](-\omega\sqrt{L'C'})^2 = -A\cos(\ )\omega^2 L'C'$$

equal so pr

$$\frac{\partial^2 v}{\partial t^2} = -A\left[\cos(\quad\right]\omega^2 \ ; \quad L'C'\frac{\partial^2 v}{\partial t^2} = -L'C'A\cos(\ )\omega^2$$

$$-\frac{\partial v(z,t)}{\partial z} = L'\frac{\partial i(z,t)}{\partial t}$$

$$+A\sin(\quad)(-\omega\sqrt{L'C'}) = L'\frac{\partial i(z,t)}{\partial t}$$

integrating both sides

$$i(z,t) = \int_t + \frac{A}{L'}\left[\sin\left(\omega t - \omega\sqrt{L'C'}\,z - \phi_0\right)\right]\left(-\omega\sqrt{L'C'}\right)\,dt$$

$$i(z,t) = +\frac{\sqrt{L'C'}}{L'}A\cos(\quad) = \sqrt{\frac{C'}{L'}}\underbrace{A\cos(\quad)}_{v(z,t)}$$

$$\boxed{\frac{v(z,t)}{i(z,t)} = \sqrt{\frac{L'}{C'}}}$$

(4)

$$-\frac{\partial \tilde{V}}{\partial z} = (R' + j\omega L')\tilde{I} \quad (a)$$

$$-\frac{\partial \tilde{I}}{\partial t} = (G + j\omega C')\tilde{V} \quad (b)$$

$$\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$$

$$\frac{\partial^2 \tilde{V}}{\partial z^2} - \gamma^2 \tilde{V} = 0$$

$$\frac{\partial^2 \tilde{I}}{\partial z^2} - \gamma^2 \tilde{I} = 0$$

$$Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$$

assume

$$\left.\begin{aligned}\tilde{V} &= V_0^+ e^{-\gamma z}\\ \tilde{I} &= I_0^+ e^{-\gamma z}\end{aligned}\right\}\text{ wave to } +z \text{ dir}$$

substitute in (a)

$$-(-\gamma)V_0^+ e^{-\gamma z} = (R' + j\omega L')I_0^+ e^{-\gamma z}$$

$$\frac{V_0^+}{I_0^+} = \frac{R'+j\omega L'}{\underbrace{\sqrt{(R'+j\omega L')(G'+j\omega C')}}_{\gamma}} = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} = +Z_0$$

assume

$$\left.\begin{aligned}\tilde{V} &= V_0^- e^{+\gamma z}\\ \tilde{I} &= I_0^- e^{+\gamma z}\end{aligned}\right\}\text{ wave in } -z \text{ direction}$$

substitute in (a)

$$-(+\gamma)V_0^+ e^{+\gamma z} = (R' + j\omega L')I_0^+ e^{+\gamma z}$$

$$\frac{V_0^-}{I_0^-} = -Z_0$$