Urman

Phasor solution (steady state)

$$\hat{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$\hat{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

sol. to wave eqn derived from telegrapher's eqns

with $Z_0 = \sqrt{\dfrac{R'+j\omega L'}{G'+j\omega C'}}$

from telegrapher's equations $\Rightarrow$ $I_0^+ = \dfrac{V_0^+}{Z_0}$, $I_0^- = -\dfrac{V_0^-}{Z_0}$

and $\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$

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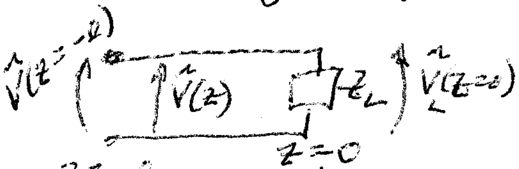
by setting $Z_L = \dfrac{\hat{V}_L(z=0)}{\hat{I}_L(z=0)}$, find $\dfrac{V_0^-}{V_0^+} = \dfrac{Z_L - Z_0}{Z_L + Z_0} \equiv \Gamma_v$

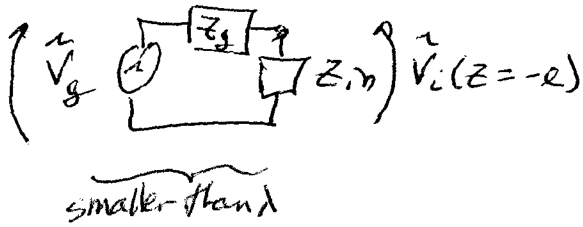


$\dfrac{I_0^-}{I_0^+} = -\Gamma_v$

and thus
$$\hat{V}(z) = V_0^+\left(e^{-\gamma z} + \Gamma_v e^{+\gamma z}\right)$$
$$\hat{I}(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_v e^{+\gamma z}\right)$$

for lossless case ($R' = G' = 0$)
$$\hat{V}(z) = V_0^+\left(e^{-j\beta z} + \Gamma_v e^{+j\beta z}\right)$$
$$\hat{I}(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma_v e^{+j\beta z}\right)$$
with $Z_0 = \sqrt{\dfrac{L'}{C'}}$; $\beta = \omega\sqrt{L'C'}$

$$Z_{in}(-\ell) = \frac{\hat{V}(z=-\ell)}{\hat{I}(z=-\ell)} = Z_0\left(\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right)$$



$$\hat{V}_i(z=-\ell) = \frac{\hat{V}_g Z_{in}}{Z_g + Z_{in}}$$
$$\hat{V}_i(z=-\ell) = V_0^+\left(e^{+j\beta\ell} + \Gamma_v e^{-j\beta\ell}\right)$$

$$V_0^+ = \left(\frac{\hat{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{+j\beta\ell} + \Gamma_v e^{-j\beta\ell}}\right)$$

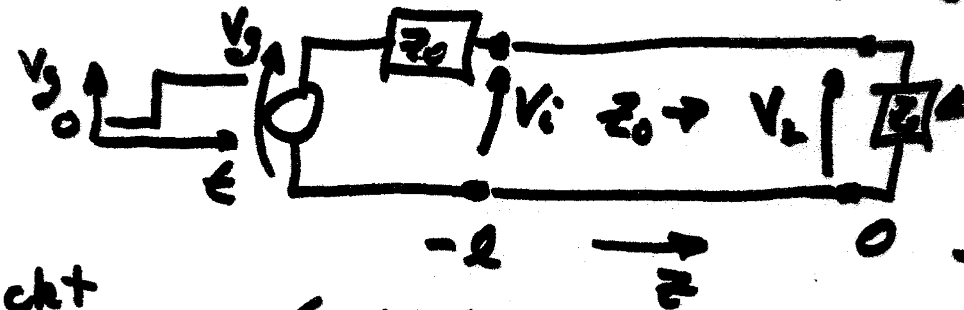
$\Gamma_v = |\Gamma_v| e^{j\theta_r}$

$$\therefore\ \hat{V}(z) = \left[\left(\frac{\hat{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta\ell} + \Gamma_r e^{-j\beta\ell}}\right)\right]\left[e^{-j\beta z} + \Gamma_r e^{+j\beta z}\right]$$
$$\hat{I}(z) = \left[\left(\quad\right)\left(\quad\right)\right]\left[\frac{1}{Z_0}\right]\left[e^{-j\beta z} - \Gamma_r e^{+j\beta z}\right]$$

St. Wr. Ratio: $S = \dfrac{1+|\Gamma_v|}{1-|\Gamma_v|} = \dfrac{|V(z)|_{max}}{|V(z)|_{min}}$; $|\hat{V}(z)| = |V_0^+|\left(1 + |\Gamma_v|^2 + 2|\Gamma_v|\cos(2\beta z + \theta_r)\right)^{1/2}$

MATCHED LINE WITH STEP FUNCTION INPUT AND LINE CHARACTERISTIC IMPEDENCE Z_0

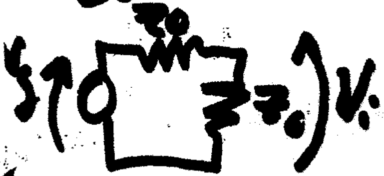
$\leftarrow \Gamma = 0$ $\quad Z_L = Z_0 \quad$ $\vec{\Gamma} = \dfrac{Z_L - Z_0}{Z_L + Z_0}$



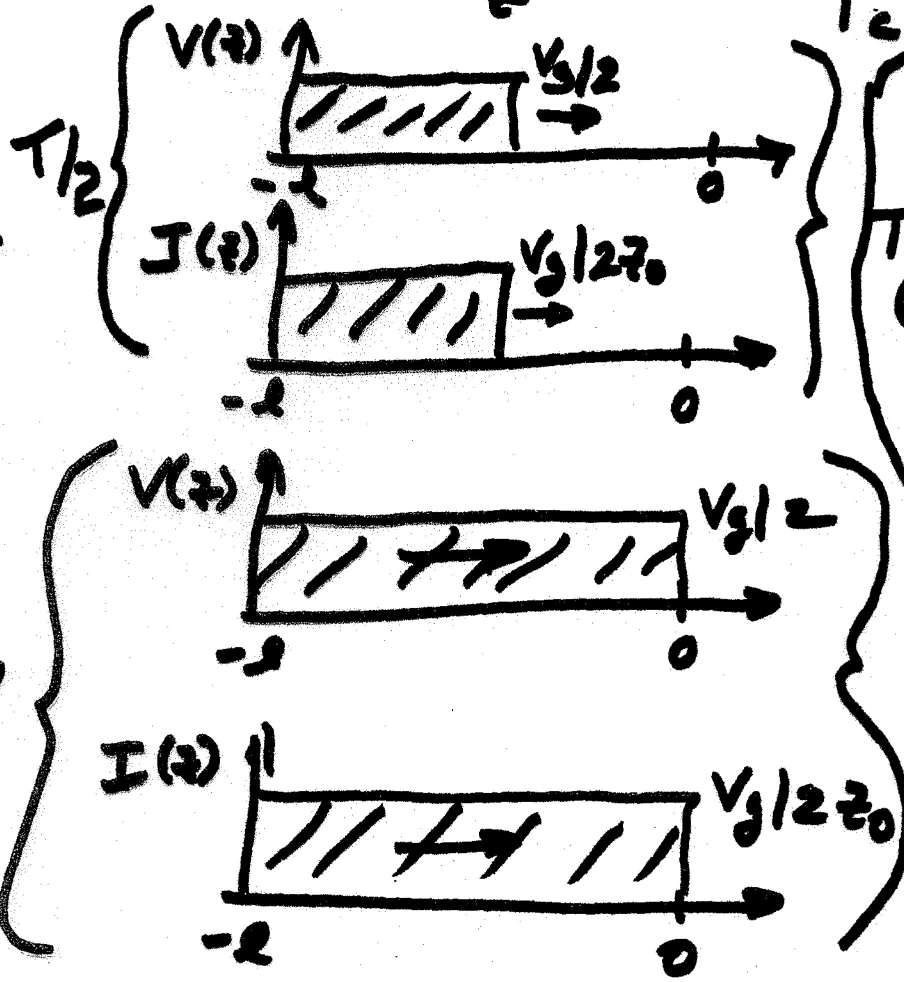
$\vec{\Gamma}_v = 0$

$\vec{\Gamma}_c = 0$

equiv ckt of source



$V_i = \dfrac{V_g}{2}$



$T \rightarrow \infty$

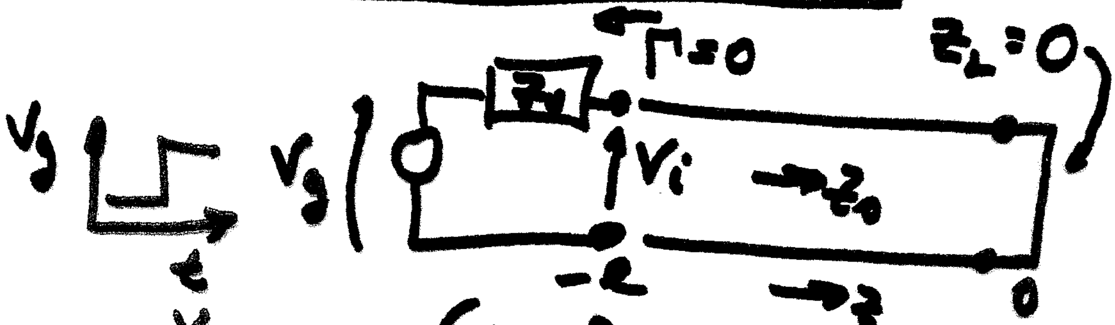
$v = \dfrac{1}{\sqrt{L'C'}}$

$T = l/v$ (prop. time on line)

in d.c. case: $V_L = V_g/2$

$I_L = V_g/2Z_0$

SHORT CIRCUIT LINE

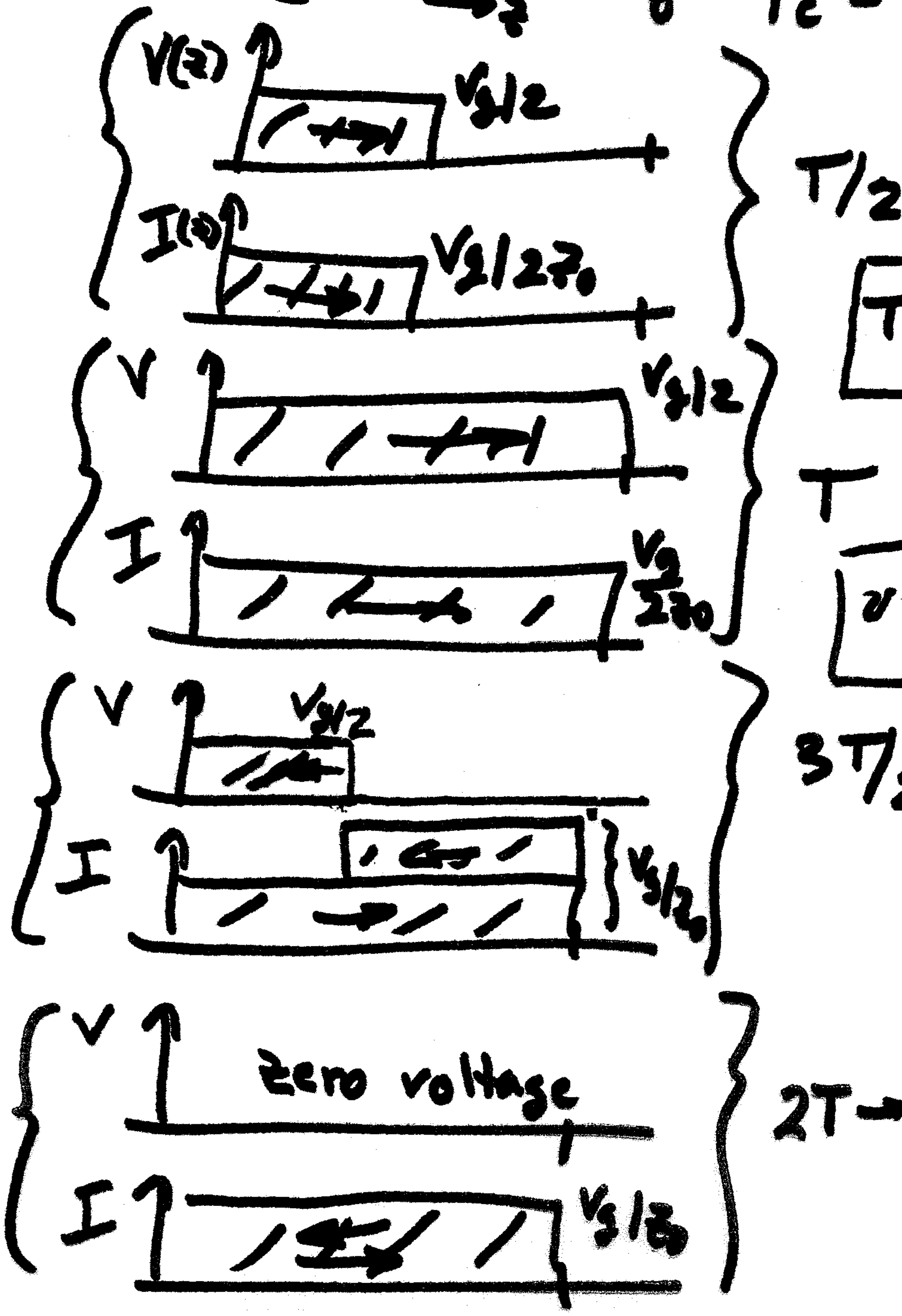


$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Gamma_V = -1$

$\Gamma_C = +1$

$V_i = \dfrac{V_g}{2}$



$\boxed{T = \frac{\ell}{v}}$

$\boxed{v = \frac{1}{\sqrt{L'C'}}}$

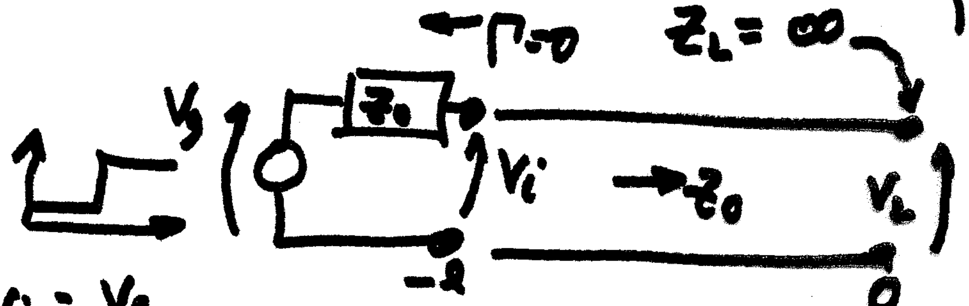
in d.c. case

$V_L = 0$

$I_L = V_g/Z_0$

OPEN CIRCUIT LINE

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$



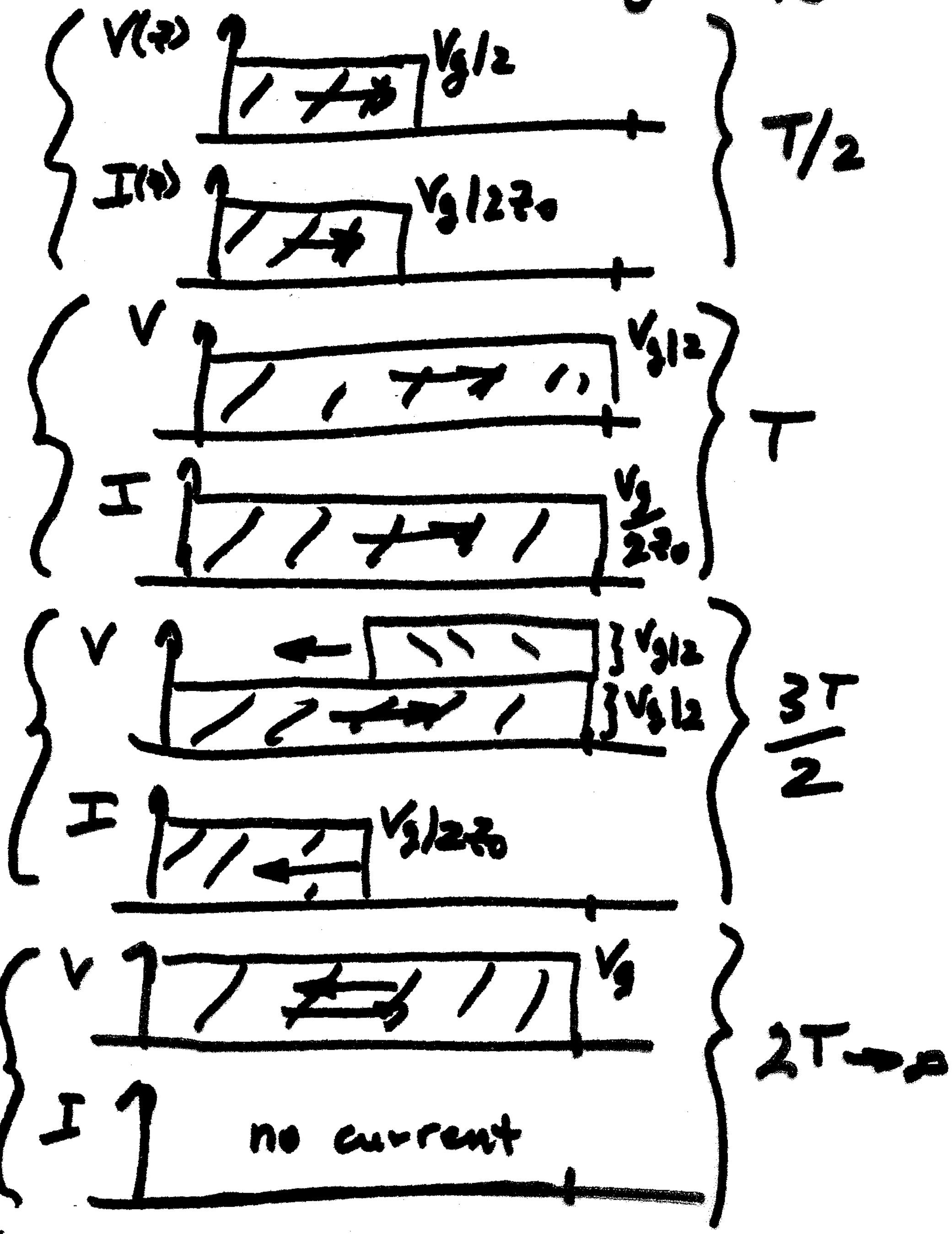
$\Gamma_v = +1$

$\Gamma_c = -1$

$V_i = \frac{V_g}{2}$

$I_i = \frac{V_0}{Z_g + Z_0}$

$V_i = \frac{V_0 Z_0}{Z_g + Z_0}$



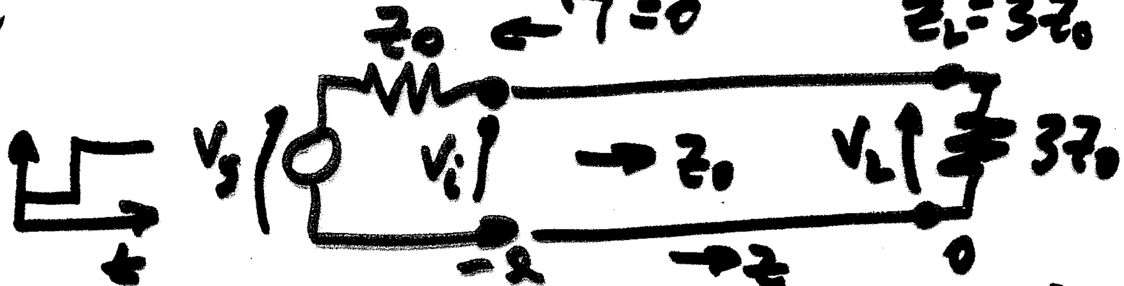
in d.c. case

$V_L = V_g$

$I_L = 0$

(also true independent of source impedance, but ∞ bounces)

LINE WITH TERMINATION: $R_L = 3Z_0$

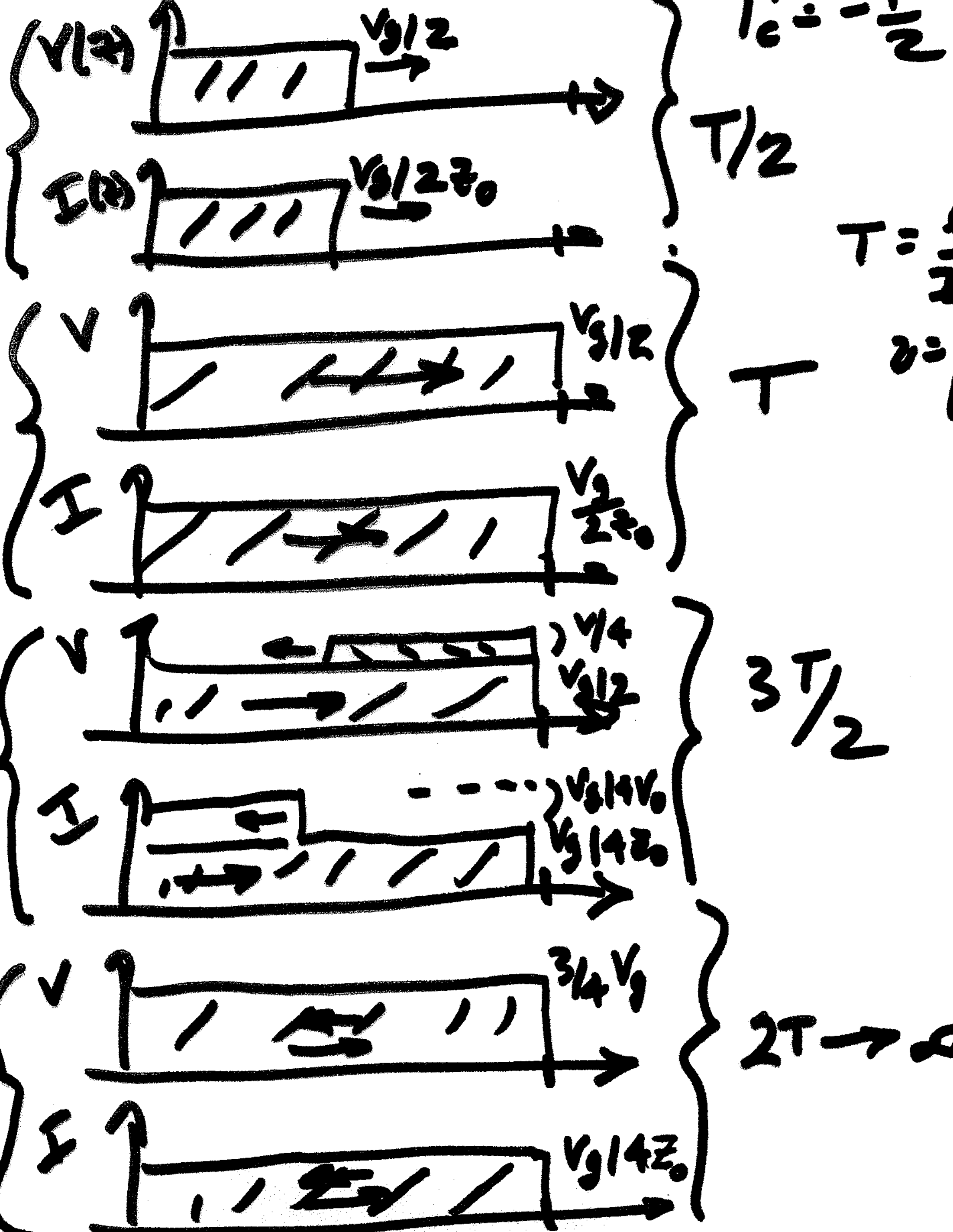


$$\Gamma_V = \frac{3Z_0 - Z_0}{3Z_0 + Z_0}$$

$$\Gamma_V = +1/2$$

$$\Gamma_C = -\frac{1}{2}$$

$$V_i = \frac{V_g}{2}$$



$$T = \frac{\ell}{v}$$

$$v = \frac{1}{\sqrt{LC'}}$$

in d.c. case

$$V_L = \frac{3Z_0}{4Z_0} V_g = \frac{3}{4} V_g$$

$$I_L = V_g/4Z_0$$